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No-scale supersymmetry breaking vacua and soft terms with torsion

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Abstract: We analyze the conditions to have no-scale supersymmetry breaking solutions of type IIA and IIB supergravity compactified on manifolds of SU(3)-structure. The supersymmetry is spontaneously broken by the intrinsic torsion of the internal space. For type IIB orientifolds with O9 and O5-planes the mass of the gravitino is governed by the torsion class $\mathcal{W}_1$, and the breaking is mediated through F-terms associated to descendants of the original $\mathcal{N} = 2$ hypermultiplets. For type IIA orientifolds with O6-planes we find two families of solutions, depending on whether the breaking is mediated exclusively by hypermultiplets or by a mixture of hypermultiplets and vector multiplets, the latter case corresponding to a class of Scherk-Schwarz compactifications not dual to any geometric IIB setup. We compute the geometrically induced μ-terms for D5, D6 and D9-branes on twisted tori, and discuss the patterns of soft-terms which arise for pure moduli mediation in each type of breaking. As for D3 and D7-branes in presence of 3-form fluxes, the effective scalar potential turns out to possess interesting phenomenological properties.

Keywords: Superstring Vacua, Supersymmetry Breaking, Supergravity Models, Flux compactifications.

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1. Introduction

Compactifications with fluxes (see [1] for reviews) have been intensively studied in the past few years for their potential phenomenological applications. They provide us with powerful tools for finding stable or metastable vacua within string theory. The topological requirement for a reduction to a four-dimensional low energy supersymmetric effective theory is that the internal manifold should allow a nowhere vanishing spinor. Six-dimensional manifolds admitting nowhere vanishing spinors have structure group reduced to SU(3) or subgroups of it [2, 3]. On manifolds of G-structure, there is always a torsionful connection under which the G-invariant spinor is covariantly constant. Torsion leads to non-integrability of the G-structure, and can be thought as another NSNS flux of the theory (and thus is sometimes called "(geo)metric flux"). In some situations it can be dual to NSNS 3-form flux H.

The effective $\mathcal{N}=1$ theory for reductions on (orientifolds of) SU(3)-structure manifolds [4 − 11] is defined by a Kähler potential and a superpotential. The former describes the space of moduli consisting of variations of the RR potentials and deformations of B-field and metric in the class of SU(3) structure manifolds. The superpotential involves all the fluxes, RR and NSNS (H and torsion). Supersymmetric vacua can be found either by varying the action [12 − 17], or directly by solving the six-dimensional internal first order differential equations for supersymmetric vacua [18]. The two procedures have been shown to be equivalent [19].

After the huge progress achieved in understanding the conditions for supersymmetric vacua and finding examples, the path continues by exploring the mechanisms of supersymmetry breaking, and finding stable or metastable non-supersymmetric vacua. Spontaneous breaking of supersymmetry can be generated dynamically, or be present already at tree level. Long-lived metastable vacua with dynamically broken supersymmetry were found in SQCD [20], and their stringy realizations proposed in [21 − 23]. On the other hand, supergravity vacua with supersymmetry broken at tree level have been mostly considered in the framework of type IIB supergravity compactified in Calabi-Yau orientifolds with O3-planes and a imaginary self-dual combination of NSNS and RR 3-form fluxes [24]. There, the amount of supersymmetry breaking is a tunable parameter which can be set to zero, and the solutions can be seen as "marginal" deformations of $\mathcal{N}=1$ vacua. From the four-dimensional point of view, the sector that breaks supersymmetry only involves moduli descending from $\mathcal{N}=2$ hypermultiplets (which in this case correspond to deformations of the Kähler form). The cosmological constant vanishes at tree level, and the supersymmetry breaking vacua are of no-scale type [25]. At the quantum level, however, non-perturbative effects may lift the remaining flat directions and restore supersymmetry in an AdS vacuum. This AdS point has been the basic building block in many of the recent attempts to address the problem of supersymmetry breaking within compactifications of string theory, and also for building up models with de-Sitter minima [26].

When D-branes are present, spontaneous breaking of supersymmetry in the bulk is communicated to the brane sector by the moduli in the closed string sector (neutral matter), and manifests in the open string sector (charged matter) as soft-breaking of supersym-

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metry [27 − 29]. Soft-supersymmetry breaking terms for D-branes on Calabi-Yau manifolds or tori in the presence of supersymmetry breaking fluxes have been obtained in [30 − 38]. These papers show that no-scale spontaneous breaking of supersymmetry by imaginary self-dual 3-form fluxes is not communicated to D3-branes. However, for branes wrapping internal dimensions, richer patterns of soft-terms arise.

Motivated by all these results, in this paper we look for classes of no-scale supersymmetry breaking vacua involving orientifolds (O5/O9 and O6) of manifolds of SU(3) structure. Supersymmetry is broken spontaneously by fluxes and torsion. The supersymmetry breaking vacua we discuss are basically divided into two types, one where the supersymmetry breaking sector lies entirely in descendants of $\mathcal{N} = 2$ hypermultiplets, and another one where the breaking sector involves the two type of moduli, those descending from hypermultiplets and those from vector multiplets. The former are believed to be T-dualizable to O3 setups of the class in [24], while the latter, which includes Scherk-Schwarz mechanism [39], seem to have non-geometric O3 duals. We illustrate each class with examples in toroidal models. Similar toroidal no-scale vacua were obtained from the four-dimensional low-energy action in [12, 15], whereas some previous work on no-scale supersymmetry breaking in supergravity compactifications was carried out in [40].

In a similar fashion than H and RR fluxes, metric fluxes induce μ-terms as well as soft-supersymmetry breaking terms on D-branes. In the article we also study the effect of torsion on D5, D9 and D6-branes in toroidal models. Using the brane superpotentials of [41], we find the torsion induced μ-terms. Finally, we analyze the soft-supersymmetry breaking patterns for the classes of supersymmetry breaking vacua discussed previously, for pure moduli mediation.

The paper is organized as follows. In section 2 we show the basic features of compactifications on manifolds of SU(3) structure. We use concepts of generalized complex geometry [42, 43], which we find best adapted to describe the bulk and brane physics. In section 3 we discuss supersymmetry breaking no-scale vacua in IIB, illustrate with examples, and find the induced μ-terms on D9 and D5-branes. In section 4 we analyze the IIA counterparts. In section 5 we study the soft-supersymmetry breaking patterns for the two classes of supersymmetry breaking mechanisms discussed in sections 3.1 and 4.1. Section 6 contains a summary and conclusions. Appendices A, B and C present some of the conventions used, as well as some technical details needed in the text.

2. Compactifications with SU(3) structure and D-branes/orientifold planes

In this section we review the necessary features about Minkowski compactifications on manifolds of SU(3) structure, and supersymmetric D-branes on them.

2.1 Type II supergravities on SU(3) structure manifolds: bulk

We study warped compactifications on manifolds of SU(3) structure, i.e. the ten-dimensional metric is given by

$$ds^2 = e^{2A}\eta_{\mu\nu}dx^\mu dx^\nu + ds^2_{\mathcal{M}_6}\,, \quad \mu = 0, 1, 2, 3 \tag{2.1}$$

where e^{2A} is the warp factor. The internal manifold $\mathcal{M}_6$ has SU(3) structure [2, 3], which we define in the following subsection, and use all throughout the text.

2.1.1 SU(3) structure definitions

On a manifold of SU(3) structure there is a globally defined SU(3) invariant non-degenerate 2-form J, and a holomorphic 3-form Ω satisfying

$$J \wedge \Omega = 0 \,, \qquad J \wedge J \wedge J = -i \frac{3}{4} \frac{\mathcal{N}_J}{\mathcal{N}_\Omega} \Omega \wedge \bar{\Omega} \,, \tag{2.2}$$

for some constants $\mathcal{N}_J$, $\mathcal{N}_\Omega$. The former conditions say that J is a (1,1)-form in the complex structure defined by Ω. The constants $\mathcal{N}_J$, $\mathcal{N}_\Omega$ define the normalization of J and Ω, in the following sense,[1]

$$\mathcal{N}_J \equiv \frac{1}{6} \int J \wedge J \wedge J \quad , \quad \mathcal{N}_\Omega \equiv \frac{1}{8i} \int \Omega \wedge \overline{\Omega} \,. \tag{2.3}$$

In this parametrization, $\mathcal{N}_J$ corresponds to the volume of the manifold whereas $\frac{1}{6}J^3$ is the volume form. A very important fact is that for manifolds with just SU(3) structure, there are no globally defined 1-forms.

J defines a symplectic structure J_{mn} (a skew-symmetric map from $T \times T$ to $\mathbb{R}$ with inverse J^{-1}) and Ω a complex structure $I^m{}_n$ (a map from T to itself that squares to -1). Provided (2.2) is satisfied, both structures intersect on a SU(3) structure. The complex structure I can be read off from the local decomposition of Ω in terms of holomorphic 1-forms z^i, namely we can write locally $\Omega = \frac{1}{6}\epsilon_{ijk} z^i \wedge z^j \wedge z^k$. The dual vectors to z^i, that we call ∂_{z^i}, form a basis for holomorphic vectors $v = v^i \partial_{z^i} = v^m \partial_m$, where ∂_m is a basis of real coordinates. The complex structure should be such that a vector constructed in this way is holomorphic, namely $I^m{}_n v^n = i v^m$.

J and I (or equivalently Ω) define a metric, given by

$$g_{mn} = J_{mp}\, I^p{}_n \,, \tag{2.4}$$

which is automatically symmetric if the first condition in (2.2) is satisfied. The SU(3) structure can be given alternatively by the metric and a globally defined SU(3) invariant spinor η. Then, J and Ω can be constructed as bilinears of the spinor, as we show in (A.2).

If the symplectic and holomorphic forms are closed, $dJ = 0$, $d\Omega = 0$, the corresponding structures are integrable. For the case of Ω, this implies that there are local functions f^i such that the 1-forms $z^i = df^i$ (i.e., the equation $z = df$ is integrable). An analogous statement can be made with integrable symplectic structures. In such case, the manifold has SU(3) holonomy. On a generic SU(3) structure, none of the structures is integrable, and therefore dJ and $d\Omega$ are not zero. The 3-form dJ and 4-form $d\Omega$ can be decomposed in SU(3) representations, and the corresponding components are the torsion classes, defined as [2]

$$dJ = \frac{3}{2} \frac{\mathcal{N}_J}{\mathcal{N}_\Omega} \mathrm{Im}(\mathcal{W}_1 \overline{\Omega}) + \mathcal{W}_4 \wedge J + \mathcal{W}_3 \,, \tag{2.5}$$

$$d\Omega = \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \overline{\mathcal{W}}_5 \wedge \Omega \,, \tag{2.6}$$

[1] A usual convention is to take the ratio $\mathcal{N}_J/\mathcal{N}_\Omega = 1$, but here we find it more convenient to leave it unfixed.

where $\mathcal{W}_1$ is a complex scalar, $\mathcal{W}_2$ a complex primitive (1,1) form, $\mathcal{W}_3$ a real primitive $(2,1)+(1,2)$ form and $\mathcal{W}_4$ and $\mathcal{W}_5$ real vectors ($\mathcal{W}_5$ is actually a complex (1,0)-form, which has the same degrees of freedom).

2.1.2 SU(3) structures and generalized complex geometry

Generalized complex geometry [42, 43] is a suitable framework for describing IIA and IIB on the same footing. We will give here just a very minimal review of it containing the basic definitions we will use. More extensive reviews in the context of flux compactifications can be found for example in [7, 44–47].

Complex and symplectic structures can actually be defined by a single type of structure: a generalized complex structure [42, 43]. Generalized complex structures are defined in an analogous way as standard complex structures, i.e. as maps from a bundle to itself that square to -1. The bundle in question is however extended (or generalized) to the sum of the tangent plus cotangent bundles of the manifold, $T_{\mathcal{M}}\oplus T^*_{\mathcal{M}}$. From J and I we can build the following generalized complex structures,

$$\mathcal{J}_- = \begin{pmatrix} I & 0 \\ 0 & -I^T \end{pmatrix} , \qquad \mathcal{J}_+ = \begin{pmatrix} 0 & J^{-1} \\ -J & 0 \end{pmatrix} , \tag{2.7}$$

where the meaning of the subscripts plus and minus will become clear later.

There is a one to one map between generalized complex structures and $O(6,6)$ pure spinors (i.e., spinors annihilated by half of the Clifford(6,6) gamma matrices). Given a pure spinor Φ, its corresponding generalized complex structure $\mathcal{J}_\Phi$ is such that the $+i$ eigenbundle of $\mathcal{J}_\Phi$ is the annihilator of the pure spinor. In addition, $O(6,6)$ spinors are isomorphic to sums of forms.[2] Positive (negative) chirality spinors are associated to even (odd) forms.

The $O(6,6)$ pure spinors corresponding to (2.7) are

$$\Phi_- = 8e^{i\theta_-}\,\eta_+\eta_-^\dagger = -ie^{i\theta_-}\left(\frac{\mathcal{N}_J}{\mathcal{N}_\Omega}\right)^{1/2}\Omega\,, \qquad \Phi_+ = 8e^{i\theta_+}\,\eta_+\eta_+^\dagger = e^{i\theta_+}e^{-iJ}\,, \tag{2.8}$$

where η is the $O(6)$ spinor defining the SU(3) structure. In order to get the forms, we have used the Fierz identity (A.4) and the bilinears in (A.2). Notice that Φ_- (Φ_+) contains only odd (even) forms, as it should be from their chiralities. This explains the use of plus and minus in (2.7). Moreover, the one to one correspondence between generalized complex structures and pure spinors define the latter up to some overall complex number, that we chose to be $8\exp(i\theta_\pm)$. These phases will be important later, and are fixed by the orientifold projection.

The action of the B-field on the pure spinors can be encoded in the "B-transform" of the spinor, $e^{-B}\Phi$, where $e^{-B}=1-B\wedge+(1/2)B\wedge B\wedge+\cdots$. It is not hard to show that if Φ is a pure spinor, then $e^{-B}\Phi$ is also pure. This allow us to work in terms of some new spinors $\tilde\Phi_\pm\equiv e^{-B}\Phi_\pm$ which define not only the metric, but also the B field. However, if Φ is closed, $\tilde\Phi$ is generically not closed, as it contains dB. It is useful to define the twisted

[2] The isomorphism is $\gamma^{m_1\ldots m_k}\cong e^{m_1}\wedge\ldots\wedge e^{m_k}$.

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exterior derivative $d_H = d - H\wedge$, where H may also contain a possible background flux $\bar{H}$, such that

$$d\tilde{\Phi} \equiv d\left(e^{-B}\Phi\right) = e^{-B} d_H \Phi \ . \tag{2.9}$$

If $\tilde{\Phi}$ is integrable with respect to d, so is Φ with respect to d_H. Note that the B-field action does not modify Φ_- since for an SU(3) structure B has to be (1,1), and therefore $B \wedge \Omega = 0$. We leave it nevertheless to include the action of H on the exterior derivative.

The B-transform of a given pure spinor is associated to the B-transform of its generalized complex structure, given by

$$\mathcal{J}^B = \mathcal{B}\mathcal{J}\mathcal{B}^{-1}\,, \qquad \mathcal{B} = \begin{pmatrix} 1 & 0 \\ -B & 1 \end{pmatrix} \ . \tag{2.10}$$

Since B is a (1,1)-form, the matrix BI is symmetric and therefore $\mathcal{J}_-^B = \mathcal{J}_-$. The generalized complex structure $\mathcal{J}_+$ on the contrary is modified to

$$\mathcal{J}_+^B = \begin{pmatrix} J^{-1}B & J^{-1} \\ -(J + BJ^{-1}B) & -BJ^{-1} \end{pmatrix} \ . \tag{2.11}$$

In what follows we will mainly work with the polyforms (2.8), although for some particular purposes, such as the moduli definitions, it will be more convenient instead to make use of $\tilde{\Phi}_\pm$.

2.1.3 Orientifold projection and $\mathcal{N} = 1$ vacua

No-go theorems for Minkowski compactifications imply that whenever fluxes are turned on, we need sources of negative charge and tension if the internal manifold is compact. We therefore study compactifications on orientifolds of manifolds of SU(3) structure, concentrating on O5/O9 compactifications of type IIB, and O6 compactifications of type IIA. The orientifold projection is the selection of even states under the combined action of the world-sheet parity Ω_{P} and an involution σ (for consistency, an additional factor of $(-1)^{F_L}$ is needed for O3/O7 and O6 projections). The involution σ should be holomorphic in IIB ($\sigma I = I$) and antiholomorphic in IIA ($\sigma I = -I$). This leads to the following action on J and Ω [48 – 50][3]

$$\begin{aligned} &\text{O6:} && \sigma J = -J\,, && \sigma\Omega = \bar{\Omega}\,, \\ &\text{O5/O9:} && \sigma J = J\,, && \sigma\Omega = \Omega\ . \end{aligned} \tag{2.12}$$

We can think of this as an action on the pure spinors as follows: the world-sheet parity operator exchanges left and right movers, which on the bispinors in (2.8) has the effect of a transposition.[4] On the forms associated to the bispinors, this transposition amounts to conjugation plus some signs, which are conveniently encoded in the operator

$$\lambda(A) = \sum_n (-1)^{[(n+1)/2]} A_n\,, \tag{2.13}$$

[3] The O6 projection allows $\sigma\Omega = e^{i\theta}\bar{\Omega}$. Here we are fixing $\theta = 0$.

[4] $\Phi_\pm$ in (2.8) should be thought as $\eta_{L+}\eta^\dagger_{R\pm}$. On a manifold of SU(3) structure there is only one globally defined spinor η, and therefore we have, up to overall normalization that we fix to 1, $\eta_{L+} = e^{i\theta_L}\eta_+\,, \eta_{R+} = e^{i\theta_R}\eta_+$. The phases in (2.8) are $\theta_\pm \equiv \theta_L \mp \theta_R$

where the subindex n denotes the degree of the form and $[\ldots]$ is the integer part. The action of σ on the forms (2.8) is therefore [8],

$$
\begin{aligned}
&\text{O6:} && \sigma\Phi_+ = \lambda(\Phi_+)\,, && \sigma\Phi_- = \lambda(\bar\Phi_-)\,, \\
&\text{O5/O9:} && \sigma\Phi_+ = -\lambda(\bar\Phi_+)\,, && \sigma\Phi_- = \lambda(\Phi_-)\,.
\end{aligned} \tag{2.14}
$$

The phases $\theta_\pm$ in (2.8) are then fixed to $\theta_+ = 0$, $\theta_- = \pi/2$ for O6, and $\theta_\pm = \pi/2$, for O5/O9.

The equations for $\mathcal{N} = 1$ Minkowski vacua in terms of $\Phi_\pm$ are [18],

$$
\begin{aligned}
d_H(e^{3A-\phi}\Phi_1) &= 0\,, \\
d_H(e^{3A-\phi}\Phi_2) &= -e^{3A-\phi} dA \wedge \bar\Phi_2 - e^{3A} * \lambda(F)
\end{aligned} \tag{2.15}
$$

where

$$
\begin{aligned}
&\text{IIA:} && \Phi_1 = \Phi_+\,, && \Phi_2 = \Phi_- \\
&\text{IIB:} && \Phi_1 = \Phi_-\,, && \Phi_2 = \Phi_+\,.
\end{aligned} \tag{2.16}
$$

The RR form F is a purely internal form related to the total ten-dimensional RR field strength,

$$
F^{(10)} = F + \mathrm{vol}_4 \wedge \lambda(*F)\,, \qquad F = \begin{cases} F_0 + F_2 + F_4 + F_6 & \text{(IIA)} \\ F_1 + F_3 + F_5 & \text{(IIB)} \end{cases} \tag{2.17}
$$

where λ is defined in (2.13), $*$ is the six-dimensional Hodge dual and the RR field strengths

$$
F_n = dC_{n-1} - H \wedge C_{n-3} + e^B \bar F \tag{2.18}
$$

satisfy the Bianchi identity $d(e^{-B}F) = 0$ in absence of localized sources.

Equations (2.15) tell us that $\mathcal{N} = 1$ Minkowski vacua require one closed pure spinor, whose parity is equal to that of the RR fluxes. The latter act as an obstruction for integrability of the real part of the other pure spinor.

2.1.4 Kähler potential, superpotential and no scale vacua

The moduli for $\mathcal{N} = 1$ compactifications arrange in chiral multiplets. The orientifold projection splits the $\mathcal{N} = 2$ vector multiplets into $\mathcal{N} = 1$ vector and $\mathcal{N} = 1$ chiral multiplets. The latter contain the scalars parameterizing variations of Φ_1. The $\mathcal{N} = 2$ hypermultiplets parameterize variations of Φ_2 (plus the dilaton and axion $B_{\mu\nu}$), paired with the axions from RR scalars. The orientifold projection keeps only the variations in the real part of Φ_2, and combines them with the surviving RR scalars, in the poly-form [8]

$$
\Pi = \mathcal{C} + i\, e^{-\phi}\, \mathrm{Re}\, \Phi_2\,, \tag{2.19}
$$

with $\mathcal{C}$ the sum of RR potentials (which have the same chirality as the form Φ_2).

The $\mathcal{N} = 1$ Kähler potential is [8]

$$
K = -\log\left[-i\int\langle\Phi_1,\bar\Phi_1\rangle\right] - 2\log\left[-i\int\langle\Phi_2,\bar\Phi_2\rangle\right] - 2\log\left(e^{-2\phi}\right) \tag{2.20}
$$

where the Mukai pairing is defined as

$$\langle A, B\rangle = (-1)^{[(n+1)/2]} A_n \wedge B_{6-n} = [\lambda(A) \wedge B]_6 \ . \tag{2.21}$$

Φ_1 and Π (or more precisely their B-transforms, $\tilde{\Phi}_1$ and $\tilde{\Pi}$) should be expanded in a basis of even or odd forms under the orientifold involution, according to the case. The moduli in $\tilde{\Phi}_1$ descend directly from their $\mathcal{N}=2$ counterparts (but only those corresponding to forms with the appropriate parity survive). As for $\tilde{\Pi}$, in order to get a Kähler moduli space, some redefinitions are needed from the $\mathcal{N}=2$ counterparts. We will give the precise definitions of the moduli in sections 3.1 and 4.1.

The superpotential for SU(3) compactifications has been computed in [7, 8] and reads simply,

$$W = \int \langle \Phi_1, d_H\, \Pi\rangle \ . \tag{2.22}$$

From (2.8) and (2.19) we observe that,

$$\text{O5/O9:} \quad \Phi_1 = \left(\frac{\mathcal{N}_J}{\mathcal{N}_\Omega}\right)^{1/2} \Omega\,, \quad \Phi_2 = i e^{-iJ}\,, \qquad d_H\Pi = F_3 + i e^{-\phi} d_H J\,, \tag{2.23}$$

$$\text{O6:} \quad \Phi_1 = e^{-iJ}\,, \qquad \Phi_2 = \left(\frac{\mathcal{N}_J}{\mathcal{N}_\Omega}\right)^{1/2} \Omega\,, \quad d_H\Pi = F + iC \mathrm{Re}\, d_H\Omega\,, \tag{2.24}$$

where $C = e^{-\phi}(\mathcal{N}_\Omega^{-1}\mathcal{N}_J)^{1/2} = e^{-\phi_{(4)}}\mathcal{N}_\Omega^{-1/2}$. Substituting in (2.20) and (2.22) then leads to the familiar expressions for type IIA and IIB,

$$\text{O5/O9:} \qquad K = -\log\left[-i\int \Omega\wedge\overline{\Omega}\right] - \log\, e^{-4\phi_{(4)}} \tag{2.25}$$

$$W = \int \Omega \wedge (F_3 + i e^{-\phi} dJ)\,, \tag{2.26}$$

$$\text{O6:} \qquad K = -\log\left[\frac{4}{3}\int J\wedge J\wedge J\right] - \log\, e^{-4\phi_{(4)}} \tag{2.27}$$

$$W = \int [e^{iJ}\wedge F]_6 + iC\int \mathrm{Re}\,\Omega \wedge (H + i dJ)\,, \tag{2.28}$$

where we have performed a partial integration in order to derive the IIA superpotential and a Kähler transformation to eliminate an extra factor $(\mathcal{N}_\Omega/\mathcal{N}_J)^{1/2}$ from the IIB superpotential. In what follows we will use the notation $G \equiv d_H\Pi$.

A generic Kähler potential K and superpotential W define a potential V equal to

$$V = e^K \left(\sum_{i,j} K^{i\bar{j}} D_i W D_{\bar{j}}\overline{W} - 3|W|^2\right) \tag{2.29}$$

where i, j run over all the $\mathcal{N}=1$ moduli, and $D_i W = (\partial_i + K_i)W$ (with $K_i = \partial_i K$). If

(i) the Kähler potential for a subset of moduli $\{\tilde{\imath}\}$ satisfies a no-scale condition [25],

$$\sum_{\tilde{\imath}} K^{\tilde{\imath}\bar{\tilde{\jmath}}}\partial_{\tilde{\imath}} K \partial_{\bar{\tilde{\jmath}}} K = 3\,, \tag{2.30}$$

(ii) there are no mixed terms $K^{\tilde{\imath}\bar{\jmath}}$ in the inverse of the Kähler metric, and

(iii) the superpotential is independent of the moduli $\tilde{\imath}$, then the negative piece $-3|W|^2$ in the potential is cancelled, and the resulting potential is positive definite,

$$V = e^K \sum_{i,j\neq\tilde{\imath}} K^{i\bar{\jmath}} D_i W D_{\bar{\jmath}}\overline{W} \ . \tag{2.31}$$

This potential has an absolute minimum at $V = 0$ when $D_i W = 0$, for all $i \neq \tilde{\imath}$. At this no-scale minimum supersymmetry is broken by the F-terms of the moduli $\tilde{\imath}$, since $D_{\tilde{\imath}} W = K_{\tilde{\imath}} W \neq 0$. We will see in sections 3.1 and 4.1 that there are several choices for the subset $\{\tilde{\imath}\}$ for the Kähler potential (2.20).

2.2 D-branes on SU(3) structure manifolds

We consider D-branes extended in 4d Minkowski space-time, wrapping an internal cycle Σ. The world-volume combination $\mathcal{F} = F - P_\Sigma[B]$ (with P_Σ the projection along Σ) should satisfy the Bianchi identity $d\mathcal{F} = -P_\Sigma[H]$. We denote a brane by the pair $(\Sigma, \mathcal{F})$.

Supersymmetric "generalized cycles" $(\Sigma, \mathcal{F})$ have to satisfy the D-flatness and F-flatness conditions. The former reads [41],

$$\mathcal{D}(\Sigma, \mathcal{F}) = P_\Sigma[e^{2A-\phi}\mathrm{Im}\,\Phi_2] \wedge e^{\mathcal{F}}|_{\mathrm{top}} = 0 \tag{2.32}$$

where $\Phi_2 = \Phi_-(\Phi_+)$ in IIA (IIB), is the non-integrable pure spinor in an $\mathcal{N} = 1$ vacuum. Notice however that (2.15) implies that $e^{2A-\phi}\mathrm{Im}\,\Phi_2$ is closed on the supersymmetric vacuum.

The F-flatness conditions, which can be derived from the superpotential (2.37) below, read

$$F_m(\Sigma, \mathcal{F}) = P_\Sigma[e^{3A-\phi}(\iota_m + g_{mn}dy^n\wedge)\Phi_1] \wedge e^{\mathcal{F}}|_{\mathrm{top}} = 0 \tag{2.33}$$

with $\Phi_1 = \Phi_+(\Phi_-)$ for IIA (IIB), the integrable pure spinor, and ι_m denotes a contraction along ∂_m.

The F-flatness conditions imply that D-branes wrap generalized complex submanifolds $(\Sigma, \mathcal{F})$ [43, 51, 52], which means that their generalized tangent bundle

$$T_{(\Sigma,\mathcal{F})} = \{v + \xi \in T \oplus T^*|_\Sigma : \iota_v \mathcal{F} = P_\Sigma[\xi]\} \tag{2.34}$$

is stable under the integrable generalized complex structure associated to Φ_1 (i.e. if we denote $\mathcal{J}_1$ this generalized complex structure, $\mathcal{J}_1 X \in T_{(\Sigma,\mathcal{F})}$, $\forall X \in T_{(\Sigma,\mathcal{F})}$). In type IIB, $\Phi_1 = \Phi_-$ is proportional to Ω, which defines a complex structure, and therefore the generalized complex submanifolds are complex submanifolds and $\mathcal{F}$ is (1,1). In type IIA, $\Phi_1 = \Phi_+$ is proportional to e^{-iJ}, which defines a symplectic structure, and therefore the complex submanifolds wrapped by D6-branes are special Lagrangian, and $\mathcal{F} = 0$. We will see this in more detail in sections 3.3 and 4.3. The D-term conditions are stability conditions for the D-brane.

The deformations of the cycle are sections of the generalized normal bundle $N_{(\Sigma,\mathcal{F})} = (T_M \oplus T^*_M)|_\Sigma / T_{(\Sigma,\mathcal{F})}$ [41]. Given a metric on the manifold, we can split $T_M = T_\Sigma + T^\perp_\Sigma$. A section of the generalized normal bundle is of the form $X_\perp = (v_\perp, a)$, where $v_\perp \in \Gamma(T_\Sigma^\perp)$ generates the deformations of the cycle Σ, while the deformations of the gauge field are $\delta\mathcal{F} = da - P_\Sigma[\iota_{v_\perp} H]$. The last term insures than under deformations of Σ, the Bianchi identity $d\mathcal{F} = -P_\Sigma[H]$ still holds. Since the tangent bundle is stable under the integrable generalized complex structure $\mathcal{J}_1$, the latter induces a natural a complex structure on $N_{(\Sigma,\mathcal{F})}$. This implies that the holomorphic generalized normal vectors, which are associated to the four-dimensional chiral fields on the brane, are $Z = (1 - i\mathcal{J})X_\perp$.

For A-branes, $\mathcal{J}_1$ corresponds to the B-transformed of the symplectic structure J, given in (2.11). The 1-form part of the holomorphic generalized normal vectors is therefore given by $(1+iBJ^{-1})\,(a + (B + iJ)v_\perp)$. (The vector part is just $-iJ^{-1}(1+iBJ^{-1})^{-1}$ times the 1-form part). Furthermore, for supersymmetric configurations the H field is zero, and therefore a represents pure gauge transformations of the world-volume field-strength. The holomorphic fields on the brane, ϕ_i, are consequently given by

$$\phi_i = [A + (B + iJ)\, v_\perp]_i\,, \quad \text{type IIA} \tag{2.35}$$

For B-branes $\mathcal{J}_1 = \mathcal{J}_-$, given in (2.7), and corresponds to an ordinary complex structure I. H is also zero for supersymmetric O5/O9 configurations. Here it is very easy to see that the holomorphic generalized tangent vectors are given by the holomorphic normal vectors and the holomorphic 1-form gauge field, namely

$$\phi_i = [(1 + i\, I^T)A]_i\,, \qquad \phi^i = [(1 - i\, I)\, v_\perp]^i\ , \quad \text{type IIB} \tag{2.36}$$

The geometrically induced μ-terms can be computed from the D-brane superpotential. For a D-brane wrapping the cycle $(\Sigma, \mathcal{F})$, the superpotential is [41],

$$W = \int_{\mathcal{B}} P_{\mathcal{B}}[e^{3A-\phi}\Phi_1] \wedge e^{\tilde{\mathcal{F}}} \tag{2.37}$$

where $(\mathcal{B}, \tilde{\mathcal{F}})$ is a chain whose boundaries are a fixed generalized cycle $(\Sigma_f, \mathcal{F}_f)$ and $(\Sigma, \mathcal{F})$. As we will see in sections 4.3, 3.3, the D-brane superpotential is holomorphic in the D-brane fields (2.35) and (2.36).

3. Type IIB compactifications with O9/O5-planes

In this section we use the Kähler potential and bulk and brane superpotentials reviewed in the previous section to find no-scale supersymmetry breaking vacua for type IIB compactifications with O9 and O5-planes, as well as the geometrically induced μ-terms on D9 and D5-branes. Backgrounds preserving this sort of supersymmetries have been described for example in [45] under the label of type C solutions. Perhaps, the best known representative is the background constructed by Chamseddine and Volkov [53], whose AdS/CFT interpretation was given by Maldacena-Nuñez [54]. Here we will consider the possible $\mathcal{N} = 0^*$ no-scale deformations of this kind of backgrounds.

3.1 No-scale vacua

Let us write again the bulk superpotential for this type of compactification, given in (2.26),

$$W = \int \Omega \wedge (F_3 + ie^{-\phi} dJ) \tag{3.1}$$

In order to extract the maximum information from it, it is convenient to decompose it into irreducible representations of the underlying SU(3)-structure. The complex 3-form $G = F_3 + ie^{-\phi} dJ$, transforming in a $\mathbf{20} = \mathbf{10} \oplus \overline{\mathbf{10}}$ of SU(3), decomposes as

$$G = G^+ + G^- \quad , \quad *_6 G^\pm = \pm i G^\pm \, , \tag{3.2}$$

and

$$\begin{aligned} G^+ &= \frac{3}{2}\frac{\mathcal{N}_J}{\mathcal{N}_\Omega} G^+_{(1)} \overline{\Omega} + G^+_{(3)} \wedge J + G^+_{(6)} \, , \\ G^- &= \frac{3}{2}\frac{\mathcal{N}_J}{\mathcal{N}_\Omega} G^-_{(1)} \Omega + G^-_{(3)} \wedge J + G^-_{(6)} \, , \end{aligned} \tag{3.3}$$

with $G^\pm_{(1)}$ a complex zero form in the $\mathbf{1}$ of SU(3), $G^+_{(3)}$ ($G^-_{(3)}$) a complex (0,1)-form ((1,0)-form) in the $\bar{\mathbf{3}}$ ($\mathbf{3}$), and $G^+_{(6)}$ ($G^-_{(6)}$) a complex primitive (2,1)-form ((1,2)-form) in the $\mathbf{6}$ ($\bar{\mathbf{6}}$). We summarize in appendix B the different representations and forms arising in the decomposition of G.

As already pointed out in section 2.1.1, on an SU(3)-structure manifold there are no globally defined 1-forms. $G^\pm_{(3)}$ and $\mathcal{W}_5$, laying in topologically trivial representations, encode information relative to the backreaction of the fluxes and branes, and in the probe limit ($A \to 0$) $F_3 \wedge J = \mathcal{W}_4 = \mathcal{W}_5 = 0$. For the moment we concentrate on this limit, and latter on we will extend the solution to the full one with finite warping.

Plugging (3.3) into (3.1), we get

$$W = \frac{3}{2}\frac{\mathcal{N}_J}{\mathcal{N}_\Omega} \int G^+_{(1)} \Omega \wedge \overline{\Omega} = 12 i \mathcal{N}_J G^+_{(1)} \, . \tag{3.4}$$

For purely imaginary self-dual (ISD) fluxes, which we will see is a required condition, $G^+_{(1)} = e^{-\phi} \mathcal{W}_1$. A non-vanishing gravitino mass therefore generically requires the torsion class $\mathcal{W}_1$ to be non-vanishing. For vacua with spontaneously broken supersymmetry, the generalized almost complex structure is thus not integrable.

For simplicity, in what follows we assume $b_-^{(1,1)} = 0$.[5] The moduli space consists in the complex structure moduli, U^k, $k = 1, \dots b_+^{(2,1)}$, the complexified Kähler deformations, T^a, defined from the expansion of the form in (2.19), which in this case is

$$\Pi = C_2 + \, ie^{-\phi} J = i T^a \omega_a \, , \qquad a = 1, \dots, b_+^{(1,1)} \, , \tag{3.5}$$

[5] $b_-^{(1,1)}$ is the number of odd (1,1) forms in the expansion in "light modes" (for more details, see [7 – 9]) In the Calabi-Yau case, this would be the number of odd harmonic (1,1)-forms. For $b_-^{(1,1)} \neq 0$, the expansion in moduli gets slightly more complicated (see [55]). We do not give it since the no-scale condition needs $b_-^{(1,1)} = 0$.

and the axio-dilaton moduli, $S = e^{-\phi}\mathcal{N}_J + iC_{\mu\nu}$. In terms of these, the Kähler potential (2.25) splits as

$$K_\Omega = -\log[8\mathcal{N}_\Omega] \quad , \quad K_J = -\log[8e^{-3\phi}\mathcal{N}_J] \quad , \quad K_S = -\log(S+S^*)\,, \tag{3.6}$$

where $\mathcal{N}_\Omega$ and $e^{-3\phi}\mathcal{N}_J$ should be understood as functions of U^k and T^a respectively.

Let us compute the F-terms coming from (3.1). These are proportional to the covariant derivatives, $D_kW \equiv \partial_k W + W\partial_k K$, with respect to the above moduli,

$$D_{U^k}W = -\int \chi_k \wedge G^-_{(6)}\,, \tag{3.7}$$

$$D_{T^a}W = 12ie^\phi G^+_{(1),a}\mathcal{N}_J\,, \tag{3.8}$$

$$D_SW = K_SW\,, \tag{3.9}$$

where the set of primitive $(2,1)$ forms χ_k is defined through,

$$\chi_k \equiv \frac{\partial_{U^k}\mathcal{N}_\Omega}{\mathcal{N}_\Omega}\Omega - \partial_{U^k}\Omega\,, \tag{3.10}$$

and we have expanded $G_{(1)}\mathcal{N}_J$ as

$$G_{(1)}\mathcal{N}_J = (G_{(1),a}\mathcal{N}_a + G_{(1)}\mathcal{N}_{J,a})T^a + \overline{F}_{(1)}\mathcal{N}_J\,, \tag{3.11}$$

with $\overline{F}_{(1)}$ the scalar component of the F_3 background, as defined in (B.4). For a purely ISD background, $G^+_{(1),a} = e^{-\phi}\partial_{T^a}\mathcal{W}_1$. In what follows we define $\mathcal{W}_{1,a} \equiv \partial_{T^a}\mathcal{W}_1$, $\mathcal{N}_{J,a} \equiv \partial_{T^a}\mathcal{N}_J$ and $\mathcal{N}_{\Omega,k} \equiv \partial_{U^k}\mathcal{N}_\Omega$ to simplify the notation.

For $\mathcal{N} = 1$ supersymmetric vacua the F-terms have to vanish, and this requires

$$\mathcal{W}_1 = F_{(1)} = 0\,, \tag{3.12}$$

$$\mathcal{W}_3 = -e^\phi *_6 F_{(6)}\,, \tag{3.13}$$

in agreement with the conditions coming from (2.15).

We may think about relaxing these conditions in order to obtain no-scale solutions with spontaneously broken supersymmetry. For that aim, as it will be clear below, it is convenient to take the internal manifold to be the fibration of a complex 2-cycle Σ_2 over a four dimensional base B. The Kähler form splits accordingly,

$$J = J_B + J_{\Sigma_2}\,, \tag{3.14}$$

with $J_B \wedge J_B \wedge J_{\Sigma_2} = 2\mathcal{N}_J$. In general, the 2-cycle may not be trivially fibered over the base B, which in terms of torsion classes means $dJ_{\Sigma_2} \neq 0$. In addition, whenever compatible with the Σ_2 fibration, the base manifold itself may have non-trivial intrinsic torsion, $dJ_B \neq 0$, as long as $dJ_B \wedge \Omega = 0$. This ensures that the superpotential does not depend on the Kähler moduli of the base, $T^{\tilde{a}}$.

Taking a non-vanishing $G^+_{(1)}$, but independent of the Kähler moduli of the fiber, T^b, leads to

$$D_S W = K_S W \,, \tag{3.15}$$

$$D_{U^k} W = -\int \chi_k \wedge G^-_{(6)} \,, \tag{3.16}$$

$$D_{T^a} W = \begin{cases} K_{T^{\tilde a}} W & \text{for } T^{\tilde a} \\ 0 & \text{for } T^b \end{cases} . \tag{3.17}$$

Therefore, imposing $D_{U^k} W = 0$, the negative piece of the scalar potential is exactly cancelled by the non-vanishing F-terms, and we get a positive definite no-scale potential of the form (2.31), where the sum runs over $i = T^b, U^k$.

In terms of the torsion classes and the 3-form RR background, the above conditions for a no-scale supersymmetry breaking vacuum read,

$$\mathcal{W}_1 = e^{\phi} F_{(1)} \,, \tag{3.18}$$

$$\mathcal{W}_3 = -e^{\phi} *_6 F_{(6)} \,, \tag{3.19}$$

where to get the first equation we have used (3.11) and the fact that $\partial_{\tilde a} W = 0$ implies $\mathcal{N}_{J,\tilde a} G^+_{(1)} = -\mathcal{N}_J G^+_{(1),\tilde a}$. G is therefore a purely ISD form.

Furthermore, $\mathcal{W}_2$ will generically be different from zero. Indeed, for G purely ISD, we can reexpress the superpotential as,

$$W = 2ie^{-\phi} \int \Omega \wedge dJ = -2ie^{-\phi} \int d\Omega \wedge J \,. \tag{3.20}$$

Decomposing J as in (3.14), taking $T^{\tilde a} \partial_{\tilde a} W = 0$ and using $\mathcal{W}_2 \wedge J \wedge J = 0$, we obtain

$$2\mathcal{W}_1 J_B \wedge J_B \wedge J_{\Sigma_2} - \mathcal{W}_2 \wedge J_B \wedge J_{\Sigma_2} = 0 \,. \tag{3.21}$$

This completely determines $\mathcal{W}_2$ in terms of $\mathcal{W}_1$, resulting in,

$$\mathcal{W}_2 = 2\mathcal{W}_1 (J_B - 2J_{\Sigma_2}) \,. \tag{3.22}$$

Some comments are in order. First, notice that the supersymmetry breaking is mediated through F-terms associated to the moduli in the expansion of Π (S and $T^{\tilde a}$). Those descend from $\mathcal{N} = 2$ hypermultiplets spanning a quaternionic manifold. The same situation occurs in conventional Calabi-Yau compactifications of type IIB with O3-planes and 3-form fluxes. More concretely, when Σ_2 is a 2-torus and $dJ_B = 0$, both kind of setups can be related by two T-dualities on Σ_2.[6] By a slight abuse of language, we will denote this type of breaking, characterized by an ISD 3-form (or more generically by an ISD polyform) with a non-vanishing SU(3) singlet component, as "no-scale quaternionic breaking". Further examples of this type of breaking will appear in section 4.1.1 for type IIA orientifolds.

Finally, let us comment on the warp factor. We have argued that the superpotential for SU(3)-structure compactifications does not contain the effects of the warping, as these

[6]For a supersymmetric version of this setup see [56].

are encoded in topologically trivial representations. Rather, they appear as corrections to the Kähler potential of the 4d effective theory [57]. Since the non-supersymmetric piece of the background is exclusively contained in the SU(3) invariant term, as can be read from (3.18), we do not expect this deformation to mix with quantities transforming in vector representations. The latter should therefore satisfy the same relations than in the supersymmetric case, given by (2.15),

$$2i\mathcal{W}_5^* = -e^{\phi}F_{(3)} = -2i\bar{\partial}A = -i\overline{\partial}\phi \ . \tag{3.23}$$

Further support to this idea comes from the analysis of the RR tadpoles. The experience with ordinary flux compactifications and open/closed string duality tells us that the backreacted geometry can be alternatively characterized by the induced charges in the bulk. The relevant piece of the ten dimensional action is [58],

$$\begin{aligned}\int C_6\wedge dF_3 &= -i\int C_6\wedge dG = \int C_6\wedge\left[\frac{3}{2}\frac{\mathcal{N}_J e^{-\phi}}{\mathcal{N}_\Omega}\left(|\mathcal{W}_1|^2 J\wedge J+\mathcal{W}_1\overline{\mathcal{W}}_2\wedge J\right)+e^{-\phi}d *_6 \mathcal{W}_3\right]\\ &= \frac{9}{2}\frac{\mathcal{N}_J e^{-\phi}|\mathcal{W}_1|^2}{\mathcal{N}_\Omega}\int C_6\wedge J_B\wedge J_B \ + \ e^{-\phi}\int C_6\wedge d *_6 \mathcal{W}_3\,,\end{aligned} \tag{3.24}$$

where we have made use of (3.22) for the last equality. The intrinsic torsion therefore induces a non-vanishing charge of D5-brane along Σ_2, and the backreacted geometry is expected to lay within the same class than the one produced by a stack of D5-branes wrapping Σ_2. The latter indeed can be shown to satisfy equation (3.23) [59, 60].

For the sake of clarity, let us now discuss a particular example of no-scale quaternionic breaking with O9/O5-planes.

3.2 Example: $K3 \times T^2$ fibration

Consider a compact nilmanifold with tangent 1-forms e^i satisfying the equations,

$$\begin{aligned} &de^1 = de^2 = de^4 = de^5 = 0 \quad , \quad de^6 = e^4\wedge e^5 \quad , \\ &de^3 = e^4\wedge e^5 - e^1\wedge e^5 + e^2\wedge e^4 \ . \end{aligned} \tag{3.25}$$

This corresponds to a T^2 fibration over a factorizable T^4 spanned by the coordinates x^1, x^2, x^4 and x^5. Notice that this set of equations is invariant under a $\mathbb{Z}_2$ discrete symmetry reversing the coordinates of the base. We take the orientifold generated by the combined action $\Omega_P\mathbb{Z}_2$, leading to a set of O5-planes with total charge of 16 (in D5-brane units) wrapping the T^2 fiber. This construction can be understood as the orbifold limit of a $K3\times T^2$ fibration (see [61] for related constructions).

The internal metric in (2.1) is given by

$$ds_6 = \frac{e^{-2A}}{u}\left(\tilde{t}_1|e^1+iue^4|^2+\tilde{t}_2|e^2+iue^5|^2\right)+\frac{e^{2A}}{u}t|e^3+iue^6|^2 \ , \tag{3.26}$$

with $\tilde{t}_i$, t the real parts of, respectively, the Kähler moduli of the base and the fiber[7] and u is the overall complex structure modulus, that is fixed to a real value in the solution (i.e.,

[7]We will always denote the real part of a field with the same letter in lowercase.

the complex structure axions are zero). Here, $e^i = dx^i$ for $i = 1, 2, 4, 5$ and

$$e^3 = dx^3 + x^4 dx^5 - x^1 dx^5 - x^4 dx^2 \,, \quad e^6 = dx^6 + x^4 dx^5 \,. \tag{3.27}$$

J and Ω are given by

$$J = J_B + J_{T^2} = -e^{-2A}(\tilde{t}_1 e^1 \wedge e^4 + \tilde{t}_2 e^2 \wedge e^5) - e^{2A} t \, e^3 \wedge e^6 \,, \tag{3.28}$$

$$\Omega = e^{-A}(e^1 + iue^4) \wedge (e^2 + iue^5) \wedge (e^3 + iue^6) \,. \tag{3.29}$$

Notice in particular that $dJ_B = 0$, as corresponds to a CY_2 manifold, and the model can be related to an ordinary T^6 orientifold with O3-planes and ISD 3-form flux by T-dualizing along x^3 and x^6.

From (3.28) and (3.29) we extract the torsion classes,

$$\mathcal{W}_1 = -\frac{e^{3A}}{6\tilde{t}_1\tilde{t}_2}(3iu + 1) \,, \tag{3.30}$$

$$\mathcal{W}_2 = -\frac{e^{3A}}{3\tilde{t}_1\tilde{t}_2}(3iu + 1)(J_B - 2J_{T^2}) \,, \tag{3.31}$$

$$\mathcal{W}_3 = \frac{e^{2A}}{8}\frac{t}{u^3}(u + i)(z^1 \wedge z^2 \wedge \bar{z}^3 + z^1 \wedge \bar{z}^2 \wedge z^3 + \bar{z}^1 \wedge z^2 \wedge z^3) + c.c \tag{3.32}$$

$$- 2dA \wedge (J_B - J_{T^2}) \,, \tag{3.33}$$

$$\mathcal{W}_4 = 0 \,, \tag{3.34}$$

$$\mathcal{W}_5 = -\partial A \,, \tag{3.35}$$

with $z^a \equiv e^a + iue^{a+3}$ the holomorphic 1-forms, and we are defining $\mathcal{N}_J$ and $\mathcal{N}_\Omega$ by (2.3) in the limit $A \to 0$. Observe that $\mathcal{W}_1$ does not depend on t, so the manifold has a suitable structure to support a no-scale solution of the type described in the previous section. For that, eqs. (3.18), (3.19) and (3.23) dictate the exact expression of the 3-form background,

$$g_s F_3 = -\frac{t}{8u^3}[(1-3iu)\Omega + (1-iu)(z^1 \wedge z^2 \wedge \bar{z}^3 + z^1 \wedge \bar{z}^2 \wedge z^3 + \bar{z}^1 \wedge z^2 \wedge z^3)] + e^{2A} *_4 d(e^{-4A}) + c.c \tag{3.36}$$

with $*_4$ the hodge star in the base, and g_s the VEV of e^ϕ, i.e. we use $e^\phi = g_s e^{2A}$. Supersymmetry is broken by the F-terms of S, T_1 and T_2, proportional to $\mathcal{W}_1$, and the cosmological constant vanishes at tree level, accordingly to the no-scale structure. Finally, the Bianchi identity for F_3 determines the charge of $D5_{T^2}$-brane induced by the flux,

$$dF_3 = \frac{e^{4A}}{2g_s\tilde{t}_1\tilde{t}_2}\left(\frac{t}{u^3}(1 + 3u^2) + e^{-2A}\nabla_B^2(e^{-4A})\right) J_B \wedge J_B \,. \tag{3.37}$$

3.3 Geometrically induced μ-terms on twisted tori in IIB

In the above compactifications, besides the flux, there are generically also D5 and D9-branes wrapping respectively complex 2-cycles and the whole space. These are required to cancel the global negative RR charge induced by the orientifold planes, whenever it is not cancelled completely by the flux. The deformations of these branes are parameterized by the holomorphic normal vectors, ϕ^i, and the holomorphic 1-form gauge fields, ϕ_i, given in equation (2.36). In a realistic compactification these would be identified with the supersymmetric partners of the matter fields, i.e. with the squarks and sleptons. The pattern of soft supersymmetry breaking terms can be thus determined by the F-terms together with the possible μ-terms for ϕ^i and ϕ_i.

The superpotential (2.37) constitutes a simple way for computing the μ-terms in a given class of compactification. For the particular case of D9 and D5-branes, it reduces to [41],

$$W_{D9} = \int \Omega \wedge \omega_3 \,, \tag{3.38}$$

$$W_{D5} = \sum_i \int_{\mathcal{B}_i} \Omega \,, \tag{3.39}$$

where ω_3 is the Chern-Simons 3-form,

$$\omega_3 = \mathrm{Tr}\left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right) \,, \tag{3.40}$$

and $\{\mathcal{B}_i\}$ the set of 3-chains generated by all possible infinitesimal deformations of the generalized complex 2-cycle which the D5-brane wraps. Alternatively, (3.38) can be anticipated by arguments of anomaly cancellation, since coupling 10d super Yang-Mills to the bulk supergravity [62–64] requires $F_3 \to F_3 + \omega_3$ in (3.1), giving rise to (3.38).

Performing the integral (3.39) requires a precise knowledge of the embedding of the D5-brane in the geometry of the internal manifold. Moreover, the zero modes of ϕ^i and ϕ_i may have non-constant profiles on the compact directions. For all this, we restrict here to the particular case of twisted tori.

A twisted torus is an homogeneous parallelizable manifold with a set of globally defined 1-forms e^a. These are not closed, but satisfy the Maurer-Cartan equations

$$de^a = \frac{1}{2} f^a_{bc} e^b \wedge e^c \,, \tag{3.41}$$

for constant f^a_{bc}. Imposing $d^2 e^a = 0$, requires the constants to satisfy Jacobi identities

$$f^a_{[bc} f^g_{d]a} = 0 \;. \tag{3.42}$$

f^a_{bc} are therefore structure constants of a Lie algebra of a group G. The twisted torus is the manifold G/Γ, where Γ is a set of discrete identifications. For $f^a_{bc} = 0$, these are of the form $x^a \cong x^a + k^a$ for some constants k^a, while in the case of nonzero structure constants some of these identifications are "twisted" (for example, if $f^3_{12} = h$ and the rest are zero, then one can identify $x^1 \cong x^1 + k^1$, $x^2 \cong x^2 + k^2$, $x^3 \cong x^3 - k^1 h x^2$).

Since twisted tori are manifolds of trivial structure, as they are parallelizable, one can globally define many SU(3) structures on them. They are defined by the following pairs of Ω and J

$$\Omega = z^1 \wedge z^2 \wedge z^3 \quad , \quad J = j_{m\bar{n}} z^m \wedge \bar{z}^n \,, \tag{3.43}$$

where $j_{m\bar{n}} = -(j_{n\bar{m}})^*$ and in the basis of holomorphic 1-forms, $z^m \equiv e^m + iU^m{}_n e^{n+3}$, $m, n = 1, 2, 3$, the metric reads $g_{m\bar{n}} = -ij_{m\bar{n}}$. For completeness, we give the torsion classes in terms of the structure constants in appendix C.

	D9	$\mathrm{D5}_1$	$\mathrm{D5}_2$	$\mathrm{D5}_3$
$u_1\mu_{11}$	$\tilde{f}^{\bar{1}}_{\bar{2}\bar{3}}$	0	$\tilde{f}^{3}_{1\bar{2}}$	$\tilde{f}^{2}_{\bar{3}1}$
$u_2\mu_{22}$	$\tilde{f}^{\bar{2}}_{\bar{3}\bar{1}}$	$\tilde{f}^{3}_{\bar{1}2}$	0	$\tilde{f}^{1}_{2\bar{3}}$
$u_3\mu_{33}$	$\tilde{f}^{\bar{3}}_{\bar{1}\bar{2}}$	$\tilde{f}^{2}_{3\bar{1}}$	$\tilde{f}^{1}_{\bar{2}3}$	0

Table 1: Supersymmetric torsion induced μ-terms for D9 and D5$_i$ branes wrapping the i-th T^2 in a factorizable twisted torus.

A stack of D9-branes wrapping the entire volume of the twisted torus will contain three complex Cartan moduli $\phi_m = A_m + iU_m{}^n A_{n+3}$. From (3.38) one may extract then the μ-terms for the light modes,

$$W_{\mu,D9} = \frac{i\mathcal{N}_\Omega}{2} g_{m\bar{o}} g_{p\bar{q}} \epsilon^{\bar{s}\bar{r}\bar{o}} f^{\bar{q}}_{\bar{s}\bar{r}} \phi^m \phi^p \,, \tag{3.44}$$

where $\phi^m = g^{m\bar{n}}\phi_{\bar{n}}$ and $\epsilon^{\bar{1}\bar{2}\bar{3}} = \epsilon_{123} = -i$.

Similarly, for a stack of D5-branes wrapping the complex 2-cycle $\Pi = a_{i\bar{j}}[z^i \wedge \bar{z}^j]$, there are two normal moduli ϕ^m, $m = 1, 2$, plus a single Cartan moduli ϕ. Performing a change of basis, $\{z^1, z^2, z^3\} \to \{\tilde{z}^1, \tilde{z}^2, \tilde{z}^3\}$, such that in the new basis $\Pi = [\tilde{z}^3 \wedge \bar{\tilde{z}}^3]$, these moduli can be identified respectively with the normal coordinates $\tilde{z}^1$ and $\tilde{z}^2$ and with the gauge bundle along $\tilde{z}^3$.

From (3.39) then we extract,

$$W_{\mu,D5} = \frac{i}{2}\epsilon_{3jk} f^k_{\bar{3}m} \phi^m \phi^j \,, \tag{3.45}$$

where the indices now refer to the new complex coordinates $\tilde{z}^i$.

The complexified structure constants defining the topology of the twisted torus can be therefore arranged according to the holomorphicity/antiholomorphicity of their indices. Thus, $f^a_{b\bar{c}}$ gives rise to μ-terms for geometric moduli of D5-branes, whereas f^a_{bc} corresponds to μ-terms for the Cartan moduli of D9-branes. On top of this, $f^{a}_{\bar{b}\bar{c}}$ controls the amount of supersymmetry breaking, as derived from (C.7).

Notice that the superpotentials (3.44) and (3.45) involve structure constants with complex indices. In writing them in terms of the ones with real indices, non-holomorphic pieces in the complex structure moduli appear. Strictly speaking, only the holomorphic terms correspond to infinitesimal supersymmetric deformations of the calibrated branes [41, 46], whereas the non-holomorphic pieces can be generically traced back to $\phi^i\phi^j$ couplings in the Kähler potential [38], giving effective contributions through the Giudice-Masiero mechanism [65] in vacua with supersymmetry spontaneously broken. We will come back to this issue in section 5. Besides, the superpotential (3.39) contains also antiholomorphic terms in the brane moduli, proportional to the structure constants $f^q_{\bar{s}\bar{r}}$. These appear when the bulk almost complex structure is not integrable, and correspond again to non supersymmetric deformations of the branes. Therefore, these terms have to be discarded.

In order to match the result for the D7-brane effective μ-term [35, 38] in vacua where a T-dual description is available, the matter fields have to be rescaled accordingly. We summarize in table 1 the torsion induced μ-terms for the different types of D-branes present in a compactification on a factorizable T^6, where the structure constants have one leg on each 2-torus. For convenience, we have introduced the "rescaled" structure constants, $\tilde{f}^I_{JK}$, defined as

$$\tilde{f}^I_{JK} \equiv \frac{2u_J u_K}{t_I} f^I_{JK} \, . \tag{3.46}$$

In terms of these, the normalized μ-terms for factorizable twisted tori are

$$W_{\mu,D9} = \frac{i}{2}\epsilon_{srp} u_q^{-1} \tilde{f}^{\bar{q}}_{\bar{s}\bar{r}} (\phi^p)^2 \, , \qquad W_{\mu,D5_p} = \frac{i}{2}\epsilon_{pjk} u_j^{-1} \tilde{f}^{k}_{\bar{p}m} (\phi^m)^2 \, . \tag{3.47}$$

Notice that the spectrum is much richer than for type IIB orientifolds with O3-planes and 3-form fluxes, where only the geometric moduli of the D7-branes can be stabilized by the fluxes [35, 66 − 68]. Concretely both the Cartan moduli of the D9-branes and the geometric moduli of the D5-branes can be lifted by the intrinsic torsion. A simple intuitive example is provided by a $D9$-brane wrapping the product $S^3 \times T^3$, with holomorphic vectors $z^m = e^m + i\hat{e}^m$. The left-invariant 1-forms of the 3-sphere satisfy

$$de^m = \frac{1}{2}\epsilon_{mno} e^n \wedge e^o \, , \tag{3.48}$$

whereas the ones in the 3-torus are closed, $d\hat{e}^i = 0$. Since $h^{(1,0)}[S^3 \times T^3] = 0$, we do not expect 4d massless zero modes coming from the gauge bundle. In fact, in terms of holomorphic vectors one has from (3.48), $f^{\bar{1}}_{\bar{2}\bar{3}} = f^{\bar{2}}_{\bar{3}\bar{1}} = f^{\bar{3}}_{\bar{1}\bar{2}} = 1/4$, and therefore all the scalars transforming in the adjoint are indeed lifted from the massless spectrum by the torsion induced μ-terms, leading to pure $\mathcal{N} = 1$ super Yang-Mills in 4d.

4. Type IIA compactifications with O6-planes

Type IIA compactifications with O6-planes have been one of the preferred setups for D-brane model building during the last decade (for a recent review see e.g. [69]). The easiness for accommodating chiral fermions in bifundamental representations without breaking $\mathcal{N} = 1$ supersymmetry, makes it the perfect framework for embedding realistic gauge theories in string theory. Recently, the possibility of adding closed string fluxes to type IIA orientifold compactifications has been also considered [70 − 72, 13], resulting in the (perturbative) stabilization of all the closed string (untwisted) moduli of the compactification [73, 15, 74]. In this section we construct no-scale supersymmetry breaking solutions of type IIA compactified on orientifolds of SU(3)-structure manifolds. We will see that the resulting possibilities turn out to be richer than for type IIB orientifold compactifications.

4.1 No scale vacua

The superpotential (2.22) specialized to IIA compactifications with O6 planes is

$$W = \int \langle e^{-iJ}, F + iC\mathrm{Re}\, d_H \Omega \rangle \equiv \int \langle e^{-iJ}, G \rangle \, , \tag{4.1}$$

where the pairing $\langle , \rangle$ is defined in (2.21), F in (2.17)–(2.18) and C is the compensator field defined below (2.24). Similarly to the type IIB case, we can decompose the "flux" $G = F + iC\mathrm{Re}\, d_H\Omega$ into ISD and IASD parts under the combined action $*\lambda$,

$$G = G^+ + G^- \,, \qquad *\lambda[G^\pm] = \pm i G^\pm \,. \tag{4.2}$$

$G^\pm$ can be decomposed in representations of SU(3) in the following way [18]

$$\begin{aligned} G^+ &= \frac{\mathcal{N}_\Omega}{\mathcal{N}_J} G^+_{(1)} e^{iJ} + G^+_{mn}\gamma^m e^{-iJ}\gamma^n + G^+_m \gamma^m \bar\Omega_3 + \tilde G^+_m \Omega_3 \gamma^m \,, \\ G^- &= \frac{\mathcal{N}_\Omega}{\mathcal{N}_J} G^-_{(1)} e^{-iJ} + G^-_{mn}\gamma^m e^{iJ}\gamma^n + G^-_m \gamma^m \Omega_3 + \tilde G^-_m \bar\Omega_3 \gamma^m \,, \end{aligned} \tag{4.3}$$

where

$$\gamma^m \Phi^\pm = (dx^m \wedge + g^{mn}\iota_n)\Phi^\pm \,, \qquad \Phi^\pm \gamma^m = \pm(dx^m \wedge - g^{mn}\iota_n)\Phi^\pm \tag{4.4}$$

for $\Phi^+(\Phi^-)$ any even (odd) form. The first terms in these expressions are singlets of the SU(3) structure, the second are in the $\mathbf{8}+\mathbf{1}$, while the last two are respectively in the $\mathbf{3}$ and $\bar{\mathbf{3}}$ representations. Each term can be obtained by an appropriate integral. For example,

$$G^+_{(1)} = \frac{i}{8\mathcal{N}_\Omega}\int \langle e^{-iJ}, G\rangle \,, \qquad G^+_{mn} = \frac{i}{32\mathcal{N}_J} J_{mp}J_{nq}\int \langle \gamma^p e^{iJ}\gamma^q, G\rangle \,. \tag{4.5}$$

We give in appendix B the expressions for the other components. Using this decomposition, the superpotential (4.1) is

$$W = -8i\mathcal{N}_\Omega G^+_{(1)} \tag{4.6}$$

In type IIA compactifications with O6-planes [72, 8], the moduli are the complexified Kähler deformations T^a from the expansion

$$B + iJ = i\, T^a\, \omega_a \,, \qquad a = 1, \ldots, b^2_- \tag{4.7}$$

and the combination of axions and complex-structure deformations encoded in the form Π in (2.19), namely

$$\begin{aligned} &\Pi = C_3 + iC\mathrm{Re}\,\Omega = (\xi^K + iC\mathrm{Re}\, Z^K)\alpha_K - (\tilde\xi_\lambda + iC\mathrm{Re}\,\mathcal{F}_\lambda)\beta^\lambda \equiv i(N^K\alpha_K - U_\lambda\beta^\lambda)\,, \\ &K = 0, \ldots, h\,, \quad \lambda = h+1, \ldots, b^3_+ - 1 \end{aligned} \tag{4.8}$$

where the integer h is basis dependent, $(\alpha_K, \beta^\lambda) \equiv (\alpha_0, \alpha_k, \beta^\lambda)$ are even 3-forms, paired symplectically with the odd 3-forms $(\alpha_\lambda, \beta^K) \in \Delta^3_-$, and $\mathcal{F}_\lambda$ the derivative of the prepotential with respect to Z^λ.

The Kähler potential for the complex structure, Kähler and axion-dilaton S is given in (2.20), and reads in this case

$$K_J = -\log[8\mathcal{N}_J]\,, \qquad K_{\Omega,S} = -2\log[C^2\mathcal{N}_\Omega]\,, \tag{4.9}$$

where $\mathcal{N}_J$ and $C^2\mathcal{N}_\Omega$ should be written in terms of the moduli T^a, N^K and U_λ. The orientifold projection selects a privileged choice of the symplectic basis in (4.8) for which $h = 0$, i.e.

$$\Pi = i(N^0\alpha_0 - U_\lambda\beta^\lambda)\,, \tag{4.10}$$

and $N^0 = S = C\mathrm{Re}\,Z^0 - i\xi^0$. In the large complex structure limit,[8] $\mathcal{F} = \frac{1}{Z^0}k_{abc}Z^aZ^bZ^c$, and the Kähler potential for S and U_λ, $\lambda = 1 \ldots b^3_+ - 1$, splits into

$$K_{\Omega,S} = -\log(S + S^*) - 2\log(\mathcal{K}_{U_\lambda})\,, \tag{4.11}$$

where $\mathcal{K}_{U_\lambda} = s^{3/2}\kappa_{\alpha\beta\rho}\tau^\alpha\tau^\beta\tau^\rho$ and $\tau^\lambda \equiv \frac{C\mathrm{Re}\,Z^\lambda}{C\mathrm{Re}\,Z^0}$ should be solved as a function of U_λ. The last piece is of the no-scale form, i.e. it satisfies (2.30) for $\{\tilde{\imath}\} = \{U_\lambda\}$.

4.1.1 No-scale quaternionic breaking

The no-scale structure of the last piece of (4.11) tells us that if the superpotential does not depend on the complex structure deformations, i.e. $\partial_{U_\lambda}W = 0$, we obtain a no-scale supersymmetry breaking vacua in the large complex structure limit by demanding $D_SW = D_{T^a}W = 0$. The moduli whose F-terms are non-zero are the ones in Π, which descend from $\mathcal{N} = 2$ hypermultiplets spanning a quaternionic manifold. Therefore this case belongs to the same class of quaternionic breaking solutions discussed in the previous section, for which G is an ISD (poly)-form. In order to make this statement more precise, let us compute the F-terms corresponding to (4.1). These result in[9]

$$D_{T^a}W = \frac{1}{\mathcal{N}_J}\int\left(\mathcal{N}_J\langle\partial_{T^a}e^{-iJ}, G\rangle - \mathcal{N}_{J,T^a}\langle e^{-iJ}, G\rangle\right) = -4\mathcal{N}_J G^-_{mn}J^{mp}J^{qn}(\omega_a)_{pq}\,, \tag{4.12}$$

$$D_SW = \int\langle e^{-iJ}, G^*\rangle = -8i\mathcal{N}_\Omega(G^-_{(1)})^*\,. \tag{4.13}$$

$$D_{U_\lambda}W = W\partial_{U_\lambda}K\,, \tag{4.14}$$

where we have already imposed $\partial_{U_\lambda}W = 0$ to compute D_SW. We therefore get a no-scale vacua if the following conditions are satisfied

$$G^-_{mn} = 0\,, \qquad G^-_{(1)} = 0\,, \qquad \int H\wedge\beta^\lambda = \int dJ\wedge\beta^\lambda = 0\,, \tag{4.15}$$

[8] This setup is mirror to large volume compactifications of IIB with O3/O7 planes in the case $b^2_- = 0$. We thank T. Grimm for pointing this out to us.

[9] The derivation of (4.12) deserves some explanation. First, in order to take the derivative with respect to the Kähler moduli T^a, defined in (4.7), we have reexpressed G as,

$$G = F + iC\mathrm{Re}\,d_H\Omega = e^B[\bar{F} + d_{\bar{H}}(e^{-B}\Pi)]\,,$$

with Π given in (4.10). Since $\langle e^{-B}\Phi, e^{-B}\Psi\rangle = \langle\Phi,\Psi\rangle$ for any Φ and Ψ, we can freely wedge both sides of the Mukai pairing in (4.1) by e^{-B}. Moreover, to get the last equality we have expressed also

$$D_{T^a}W = -i\int\langle\omega_a e^{-iJ}, G\rangle + i\frac{\mathcal{N}_\Omega}{\mathcal{N}_J}\int\langle\omega_a e^{-iJ}, e^{iJ}\rangle G^+_{(1)}\,.$$

Using (4.4) then we can write

$$\omega_a\Phi^+ = \frac{1}{2}(\omega_a)_{mn}dx^m\wedge dx^n\Phi^+ = \frac{1}{4}[\gamma^m,\{\gamma^n,\Phi^+\}]\,,$$

and finally, using the bispinor expression for e^{-iJ} given in (2.8), the relation between the Mukai pairing and the norm of bispinors (A.3), the decomposition (4.3) and the bilinears (A.2), we arrive to the expression (4.12).

where the last two are required to get $\partial_{U_\lambda}W = 0$. These conditions imply that all NS fluxes (H_3 plus torsion) are determined in terms of the dilaton, the RR singlet fluxes and $F_2^{(8)}$ (see definitions in appendix B),

$$\begin{aligned}
&\mathrm{Re}\,\mathcal{W}_1 = \frac{1}{6}e^{\phi^{(4)}}\mathcal{N}_\Omega^{1/2}F_2^{(1)}\,, && H^{(1)} = \frac{1}{3}\frac{\mathcal{N}_J}{\mathcal{N}_\Omega}F_0\,, && F_4^{(1)} = F_6^{(1)} = 0\,,\\
&\mathrm{Im}\,\mathcal{W}_2 = 0\,, && \mathrm{Re}\,\mathcal{W}_2\wedge J = -e^{\phi^{(4)}}\mathcal{N}_\Omega^{1/2} * F_2^{(8)}\,, && F_4^{(8)} = 0\,,\\
&H^\lambda = -\frac{1}{2}\frac{\mathcal{N}_J}{\mathcal{N}_\Omega^{1/2}}e^{\phi^{(4)}}F_0\,\mathrm{Im}\,Z^\lambda\,, && H_0 = \frac{1}{\mathrm{Re}\,Z^0}H^\lambda\,\mathrm{Re}\,(\mathcal{F}_\lambda)\,, &&\\
&\mathcal{W}_3^\lambda = -\frac{1}{4}\frac{\mathcal{N}_J}{\mathcal{N}_\Omega}F_2^{(1)}\,\mathrm{Im}\,Z^\lambda\,, && (\mathcal{W}_3)_0 = \frac{1}{\mathrm{Re}\,Z^0}\mathcal{W}_3^\lambda\,\mathrm{Re}\,(\mathcal{F}_\lambda)\,, &&
\end{aligned} \tag{4.16}$$

where we have expanded $H^{(6)} = H^\lambda\alpha_\lambda + H_0\beta^0$, and similarly for $\mathcal{W}_3$. Notice that the difference with respect to the supersymmetric solution is precisely the singlets (while the **8** component has the same form as the supersymmetric one, analogously to the **6** in type IIB). Moreover, following the same arguments as in type IIB, we expect the warp factor to behave as in the supersymmetric solution, namely

$$2i\mathcal{W}_5^* = -e^\phi F_2^{(3)} = 2i\bar{\partial}A = \frac{2}{3}i\bar{\partial}\phi\ , \tag{4.17}$$

and $\mathcal{W}_4 = 0$, so from (4.16) and (4.17) we see that G is indeed an ISD form.

Inspired by the type IIB no-scale solutions with O5-planes of previous sections, we may also consider a slightly different class of solutions, on which K_{U_λ} splits as

$$K_{U_\lambda} = -\log(U+U^*) + K'_{U_{\tilde\lambda}} \quad , \quad K'^{\tilde\lambda\bar{\tilde\rho}}K'_{\tilde\lambda}K'_{\bar{\tilde\rho}} = 2\ . \tag{4.18}$$

This will be the case for example for twisted tori, where U is made out of the real complex structure of one of the T^2 and its axion partner. A no scale solution arises if $\partial_S W = \partial_{U_{\tilde\lambda}}W = 0$, for $U_{\tilde\lambda} \neq U$, and $D_{T^a}W = D_U W = 0$. Notice that the F-terms in this case read,

$$D_U W = \int\langle e^{-iJ}, G^*\rangle = -8i\mathcal{N}_\Omega(G^-_{(1)})^*\,,\quad D_S W = W\partial_S K\,,\quad D_{U_{\tilde\lambda}}W = W\partial_{U_{\tilde\lambda}}K\ \ (U_{\tilde\lambda}\neq U)\,, \tag{4.19}$$

with $D_{T^a}W$ still given by the first line of (4.12). Thus, we get back again the conditions (4.15), with λ now running over all $U_{\tilde\lambda}$, and $H\wedge\alpha_0 = dJ\wedge\alpha_0 = 0$. The breaking is again mediated by the $\mathcal{N}=1$ scalars descending from the $\mathcal{N}=2$ hypermultiplets, the only difference being the particular directions of the quaternionic space which enter the breaking.

4.1.2 No-scale mixed breaking: Scherk-Schwarz breaking

Apart from the no-scale solutions with the supersymmetry spontaneously broken by moduli in Π (complex structure and dilaton), in geometric type IIA compactifications with SU(3) structure there is another class of solutions on which the breaking involves also F-terms associated to the moduli in $\Phi_1 = \Phi_+$, descending from $\mathcal{N}=2$ vector multiplets. These solutions are therefore not dual to the quaternionic breaking solutions, and we believe their type IIB counterparts correspond to non-geometric compactifications. The existence of these solutions was noticed in [15] from the four dimensional point of view, however the ten dimensional construction was missing. Here, we will show that they are related to non-supersymmetric Scherk-Schwarz compactifications [39].

Indeed, consider the internal manifold to be a trivial T^2 fibration over a base $\mathcal{B}$, i.e. $\mathcal{M} = T^2 \times \mathcal{B}$, so that the Kähler form is decomposed as $J = J_{T^2} + J_{\mathcal{B}}$, with $dJ_{T^2} = 0$, and K_{U_λ} satisfying (4.18). Let us take vanishing fluxes, F=H=0, and therefore the setup is of Scherk-Schwarz type. The superpotential becomes

$$W = \int \langle e^{-iJ_{\mathcal{B}}}, iC\mathrm{Re}\, d\Omega\rangle \ . \tag{4.20}$$

Since it is independent of the Kähler modulus of the T^2 fibration, $\tilde{T}$, it leads to a no-scale structure when $J_{\mathcal{B}} \wedge \partial_{U_\lambda}(d\Omega) = 0$, for $U_\lambda \neq \tilde{U}$. Notice also that $\mathrm{Re}\, G = 0$, so G is a pure imaginary (poly)-form and the IASD SU(3) components are therefore automatically determined by the ISD components, $G^+_{(1)} = G^-_{(1)}$ and $G^+_{mn} = (G^-_{mn})^T$.

Computing the F-terms,

$$D_S W = \frac{i}{2}\int \left\langle e^{-iJ_{\mathcal{B}}}, d\left(\alpha_0 + \frac{\tilde{u}}{s}\beta_{\tilde{U}}\right)\right\rangle \ , \tag{4.21}$$

$$D_{T^a} W = \begin{cases} W\partial_{\tilde{T}}K & \text{for } T^a = \tilde{T} \\ 32i\mathcal{N}_J G^-_{mn} J^{mp}_{\mathcal{B}} J^{qn}_{\mathcal{B}} (\omega_a)_{pq} & \text{for } T^a \neq \tilde{T} \end{cases} \ , \tag{4.22}$$

$$D_{U_\lambda} W = \begin{cases} W\partial_{U_\lambda}K & \text{for } U_\lambda \neq \tilde{U} \\ -\frac{s}{\tilde{u}} D_S W & \text{for } U_\lambda = \tilde{U} \end{cases} \ , \tag{4.23}$$

we get that in order to have $D_{T^a}W = D_S W = 0$, for $T^a \neq \tilde{T}$, $G^\pm_{mn}$ has to vanish along the directions of $\mathcal{B}$ and,

$$\int J_{\mathcal{B}} \wedge d\left(\alpha_0 + \frac{\tilde{u}}{s}\beta_{\tilde{U}}\right) = 0 \ . \tag{4.24}$$

Notice that in some sense S and $\tilde{U}$ behave as a single modulus of the compactification. This will be made more explicit in a concrete example in next section.

4.2 Examples

4.2.1 No-scale quaternionic breaking

We consider here a representative of the first class of no-scale vacua discussed above, i.e. those on which the supersymmetry is spontaneously broken by F-terms associated exclusively to the $\mathcal{N} = 1$ fields descending from $\mathcal{N} = 2$ hypermultiplets (S and U_λ). These IIA solutions are mirror to the usual no-scale solutions of type IIB with O3-planes, or T-dual to the ones with O5-planes discussed in previous sections. This particular example corresponds to the ten dimensional realization of one of the no-scale vacua considered in [15].

We take the internal manifold to be a compact S^1 fibration over T^5, with O6-planes wrapping x^1, x^2, x^3 and

$$de^1 = de^2 = de^4 = de^5 = de^6 = 0 \quad , \quad de^3 = -e^4 \wedge e^5 \ . \tag{4.25}$$

To avoid cluttering, let us first give the solution in the limit $A \to 0$ and then comment on how to introduce the warp factor. Choosing,[10]

$$\Omega = (e^1+i\tau^1 e^4)\wedge(e^2+i\tau^2 e^5)\wedge(e^3+i\tau^3 e^6)\,, \quad J = -t_1 e^1\wedge e^4 - t_2 e^2\wedge e^5 - t_3 e^3\wedge e^6\,, \tag{4.26}$$

we get, $d(\mathrm{Re}\,\Omega) = e^1\wedge e^4\wedge e^2\wedge e^5$. The torsion classes are

$$\begin{aligned}\mathcal{W}_1 &= \frac{1}{6t_1t_2} \quad , \quad \mathcal{W}_2 = \frac{1}{3t_1t_2}\left(J + i\frac{3t_3}{2\tau^3} z^3\wedge\bar{z}^3\right)\,,\\ \mathcal{W}_3 &= -\frac{it_3}{8\tau^1\tau^2\tau^3}(z^1\wedge z^2\wedge\bar{z}^3 + z^1\wedge\bar{z}^2\wedge z^3 + \bar{z}^1\wedge z^2\wedge z^3) \;+\; \text{c.c.}\end{aligned} \tag{4.27}$$

where $z^a = e^a + i\tau^a e^{a+3}$. On top of this, we parameterize a possible expectation value of the NSNS 3-form as,

$$H = m\frac{t_1t_2t_3}{s} e^4\wedge e^5\wedge e^6\;. \tag{4.28}$$

This, together with the ISD condition, determines G as,

$$G = -m - \frac{st_3}{t_1t_2} e^3\wedge e^6 + ise^1\wedge e^4\wedge e^2\wedge e^5 + imt_1t_2t_3 e^1\wedge e^2\wedge e^3\wedge e^4\wedge e^5\wedge e^6\;, \tag{4.29}$$

from which we read the expectation values of the RR field strengths,

$$F_0 = -m\,, \quad F_2 = -\frac{st_3}{t_1t_2} e^3\wedge e^6\,, \quad F_4 = F_6 = 0\,, \tag{4.30}$$

and the Bianchi identity,

$$dF_2 = \frac{st_3}{t_1t_2} e^4\wedge e^5\wedge e^6 = \delta_{D6/O6}\;. \tag{4.31}$$

In terms of SU(3) representations, the only non-vanishing components of G are,

$$G^+_{(1)} = -\frac{t_3(mt_1t_2+is)}{4\tau^1\tau^2\tau^3}\,, \qquad G^+_{m\bar{n}} = \frac{1}{4}\frac{\tau^1\tau^2\tau^3}{t_1t_2t_3}\begin{pmatrix} (G^+_{(1)})^*\frac{t_1}{\tau^1} & 0 & 0\\ 0 & (G^+_{(1)})^*\frac{t_2}{\tau^2} & 0\\ 0 & 0 & G^+_{(1)}\frac{t_3}{\tau^3}\end{pmatrix} \tag{4.32}$$

Notice in particular the independence of $\mathcal{N}_\Omega G^+_{(1)}$ (i.e., of the superpotential) on the complex structure moduli, accordingly with the no-scale structure.

One can make contact with the results of [15] by decomposing the field-strengths between the Chern-Simons couplings and the background field. Indeed, from (2.18) we see that the VEV's for the axionic parts of the Kähler moduli and S are fixed as,

$$\mathrm{Im}\,T_a = \frac{1}{m}\int\left(F_2 - \overline{F}_2\right)\wedge\tilde{\omega}_a\,, \quad \mathrm{Im}\,S = \int\left(\overline{F}_4 + \frac{1}{2m}\overline{F}_2\wedge\overline{F}_2\right)\wedge e^3\wedge e^6\,, \tag{4.33}$$

in agreement with the results of [15].

[10] In terms of $\mathcal{N}=1$ moduli, $C\mathrm{Re}\,\Omega = \frac{e^{-\phi_{(4)}}}{\sqrt{\tau^1\tau^2\tau^3}}(e^1\wedge e^2\wedge e^3 - \tau^1\tau^2 e^4\wedge e^5\wedge e^3 - \tau^1\tau^3 e^4\wedge e^2\wedge e^6 - \tau^2\tau^3 e^1\wedge e^5\wedge e^6) \equiv se^1\wedge e^2\wedge e^3 - u_3 e^4\wedge e^5\wedge e^3 - u_2 e^4\wedge e^2\wedge e^6 - u_1 e^1\wedge e^5\wedge e^6$.

As argued in the previous sections, the warp factor behaves like in the supersymmetric case, i.e. the warped solution is obtained by making the replacement $e^a \to e^A e^a$, $e^{a+3} \to e^{-A} e^{a+3}$, $a = 1, 2, 3$, in (4.26). The torsion classes $\mathcal{W}_1$, $\mathcal{W}_2$ and $\mathcal{W}_3$ in (4.27), as well as H in (4.28) get multiplied by a factor of e^{3A}. Additionally, $\mathcal{W}_2$ gets a term of the form $-2i\epsilon_{ijk}\frac{\tau^i}{t_i}\partial_{\bar{i}} A\,\bar{z}^j \wedge z^k$, and F_2 a term $s\,e^A *_3 d(e^{-4A})$ (where $*_3$ is the Hodge dual on the 3-dimensional subspace 456). Besides, $\partial\phi = 3\mathcal{W}_5 = 3\partial A$, as expected from (4.17) while $\mathcal{W}_4$ is zero. Finally, the Bianchi identity (4.31) gets an additional term $-2se^{-2A}\nabla^2(e^{-4A})e^4 \wedge e^5 \wedge e^6$.

4.2.2 No scale mixed breaking

Here we consider a representative example of this class of solutions, based on an algebraic solvmanifold with,

$$\begin{aligned} &de^1 = de^4 = 0\,, \\ &de^2 = e^6 \wedge e^4 \quad , \quad de^5 = e^3 \wedge e^4\,, \\ &de^3 = e^4 \wedge e^5 \quad , \quad de^6 = e^4 \wedge e^2\,, \end{aligned} \tag{4.34}$$

As shown in [47], this solvmanifold admits a flat metric,[11] a lattice Γ such that the quotient G/Γ is compact, and O6-planes spanning the directions 123, 156, 426, 453, 125 and/or 136. The moduli space is composed of two Kähler moduli, T_1 and T_2, two complex structure moduli, U_2 and U_3, and a single axio-dilaton S.[12] In terms of these, J and $\mathrm{Re}\,\Omega$ read (again in the limit $A \to 0$)

$$\begin{aligned} J &= -t_1 e^1 \wedge e^4 \;-\; t_2(e^2 \wedge e^5 + e^3 \wedge e^6)\,, \\ \mathrm{Re}\,\Omega &= e^1 \wedge e^2 \wedge e^3 \;- e^1 \wedge e^5 \wedge e^6 - \; e^4 \wedge \left(\frac{u_2}{s}\, e^2 \wedge e^6 + \frac{u_3}{s}\, e^5 \wedge e^3\right)\,, \end{aligned} \tag{4.35}$$

and hence, for H and all the RR forms vanishing, G is given by

$$G = -2ise^1 \wedge e^4 \wedge (e^2 \wedge e^5 + e^3 \wedge e^6)\ . \tag{4.36}$$

In terms of SU(3) components,

$$G^+_{(1)} = G^-_{(1)} = \frac{ist_2}{2\tau}\,, \qquad G^+_{m\bar{n}} = G^-_{\bar{m}n} = -\frac{s}{8t_2\tau}\begin{pmatrix} i & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\,, \tag{4.37}$$

with $\tau = \sqrt{u_2 u_3}/s$ the complex structure parameter of the 2-torus spanned by e^1 and e^4. Notice that $\mathcal{N}_\Omega G_{(1)}$ is independent of U_2, U_3 and T_1. On the other hand, the F-terms associated to S and T_2 automatically vanish, thus leading to a no-scale structure with the corresponding axions stabilized as,

$$\mathrm{Im}\,S = \int \overline{F}_4 \wedge (e^2 \wedge e^5 + e^3 \wedge e^6)\,, \quad \mathrm{Im}\,T_2 = \int \overline{H} \wedge e^4 \wedge (e^5 \wedge e^6 - e^2 \wedge e^3)\ . \tag{4.38}$$

[11] Changing the sign of f^3_{45} and f^5_{34}, it admits also supersymmetric backgrounds without flux.

[12] The solution requires $u_1 = s$ (i.e. $\tau_2\tau_3 = 1$) and $t_2 = t_3$. The first condition guarantees that (4.24) is satisfied, whereas the second one implies that $G^\pm_{mn}$ takes non-zero values only along the directions of e^1 and e^4.

The torsion classes are

$$\mathcal{W}_1 = -\frac{2}{3t_1t_2}\,, \quad \mathcal{W}_2 = -\frac{2}{3t_1t_2}[2t_1e^1\wedge e^4 - t_2(e^2\wedge e^5 + e^3\wedge e^6)]\,,$$
$$\mathcal{W}_3 = \frac{i}{2\tau}t_2\bar{z}^1\wedge z^2\wedge z^3 \;+\; \text{c.c.} \tag{4.39}$$

Note that this solution does not have RR flux or H. Therefore, the orientifold planes are not needed to cancel tadpoles. However, without the orientifold projection the moduli would be those of $\mathcal{N}=2$. We expect this background to be a no-scale supersymmetry breaking solution also without the orientifolds, since the equations of motion should not be sensible to the projection. In any case, if there are orientifold planes and consequently D6-branes to cancel the tadpoles, but such that these are not on top of each other, the warp factor should behave as in the previous example.

4.3 Geometrically induced μ-terms on twisted tori in IIA

In type IIA on SU(3) structure manifolds, $\Phi_{1,2}$ are given respectively by $\Phi_{+,-}$ in (2.8). $\theta_+ = \theta_- - \pi/2$, where the phase θ_+ is a choice, and determines the location of the O6-planes. Choosing $\theta_- = \pi/2$, the orientifold projection acts as $\sigma(\Omega)=\bar{\Omega}$. Let us use real 1-forms $X^i, Y^{\hat{\imath}}$, $i,\hat{\imath}=1,2,3$, where the orientifold projection acts as $\sigma(X^i,Y^{\hat{\imath}}) = (X^i,-Y^{\hat{\imath}})$. The complex 1-forms Z^i and the symplectic form are given by

$$Z^i = X^i + i\,\tau^i{}_{\hat{\jmath}}Y^{\hat{\jmath}}\,, \qquad J_c = B + iJ = -i\,T_{i\hat{\jmath}}\,X^i\wedge Y^{\hat{\jmath}} \tag{4.40}$$

where $\tau^i{}_{\hat{\jmath}}$ are real, and $T_{i\hat{\jmath}}$ are complex Kähler moduli. These define an SU(3) structure if the matrix $T\tau^{-1}$ is symmetric, i.e $T_{i\hat{\jmath}}(\tau^{-1})^{\hat{\jmath}}{}_k = T_{k\hat{\jmath}}(\tau^{-1})^{\hat{\jmath}}{}_i$. In that case, $B+iJ$ is (1,1) with respect to the complex structure. A basis of 3-forms is given by

$$\alpha_0 = X^1\wedge X^2\wedge X^3\,, \qquad \beta^0 = Y^1\wedge Y^2\wedge Y^3\,.$$
$$\alpha_j{}^{\hat{\imath}} = \frac{1}{2}\epsilon_{jkl}X^k\wedge X^l\wedge Y^{\hat{\imath}}\,, \qquad \beta^i{}_{\hat{\jmath}} = -\frac{1}{2}\epsilon_{\hat{\jmath}\hat{k}\hat{l}}Y^{\hat{k}}\wedge Y^{\hat{l}}\wedge X^i\,. \tag{4.41}$$

The holomorphic 3-form Ω is given in this basis by

$$\Omega = \alpha_0 + i\alpha_j{}^{\hat{\imath}}\tau^j{}_{\hat{\imath}} + \beta^i{}_{\hat{\jmath}}(\mathrm{cof}\tau)_i{}^{\hat{\jmath}} - i\beta^0(\det\tau)\,, \tag{4.42}$$

where

$$(\mathrm{cof}\tau)_i{}^{\hat{\jmath}} = (\det\tau)\tau^{-1,\mathrm{T}} = \frac{1}{2}\,\epsilon_{ikm}\epsilon^{\hat{\jmath}\hat{p}\hat{q}}\tau^k{}_{\hat{p}}\tau^m{}_{\hat{q}}\,. \tag{4.43}$$

A supersymmetric D6-brane has to satisfy the D-flatness condition (2.32), which reads in this case

$$P_\Sigma(\mathrm{Im}\,\Omega) = 0\,. \tag{4.44}$$

The cycles that satisfy it are Σ^0, $\Sigma^{\hat{\jmath}}{}_i$, dual respectively to the even left invariant forms α_0 and $\beta^i{}_{\hat{\jmath}}$. The F-flatness condition (2.33) implies

$$P_\Sigma[J] = 0\,, \qquad \mathcal{F} = 0\,, \tag{4.45}$$

i.e. the branes wrap special Lagrangian submanifolds. The first condition is satisfied automatically on the cycle Σ^0, while for the cycles $\Sigma_i{}^{\hat{\jmath}}$ they impose $T_{i\hat{k}} = T_{i\hat{l}} = 0$, where $\hat{k}, \hat{l} \neq \hat{\jmath}$. This means that for a given i, there's only one $\hat{\jmath}$ such that $\Sigma_i{}^{\hat{\jmath}}$ is supersymmetric, namely $\hat{\jmath}$ is defined by the combination $T_{i\hat{m}} y^{\hat{m}}$. There are therefore in total four supersymmetric cycles, Σ^0 and Σ^i.

The superpotential (2.37) for a D6-brane wrapping $(\Sigma, \mathcal{F})$ is

$$W = \frac{1}{4} \int_{\mathcal{B}_i} e^{3A-\phi} (\tilde{F} - J_c)^2 \tag{4.46}$$

For the cycle Σ^0, dual to α_0, $\mathcal{B}_i$ are chains dual to the forms $X^1 \wedge X^2 \wedge X^3 \wedge Y^{\hat{\jmath}}$. The holomorphic brane fields are given in (2.35), and their superpotential is

$$W_{D6_0} = \frac{1}{4} \epsilon^{ikl} (T_{k\hat{r}} f^{\hat{r}}_{\hat{\imath}l} + T_{r\hat{\imath}} f^{r}_{lk}) T_{i\hat{\jmath}} \phi^{\hat{\imath}} \phi^{\hat{\jmath}} \,, \qquad \phi_i = A_i - i\, T_{i\hat{\jmath}} y^{\hat{\jmath}} \equiv T_{i\hat{\jmath}} \phi^{\hat{\jmath}} \tag{4.47}$$

For the cycle Σ_i, dual to $\beta^i{}_{\hat{\jmath}} = -\frac{1}{2}\, \epsilon_{\hat{\jmath}\hat{k}\hat{l}} Y^{\hat{k}} \wedge Y^{\hat{l}} \wedge X^i$ the superpotential is

$$\begin{aligned} W_{D6_{\mathrm{i}}} = &- \frac{1}{2} \left(T_{p\hat{l}} f^{p}_{\hat{k}\hat{\jmath}} + 2 T_{p(\hat{k}} f^{p}_{\hat{\jmath})\hat{l}} \right) T_{i\hat{\jmath}}\, \phi^{\hat{\jmath}} \phi^{\hat{\jmath}} - \frac{1}{2} \left(T_{p\hat{k}} f^{p}_{ib} + T_{(i|\hat{r}} f^{\hat{r}}_{b)\hat{k}} \right) T_{a\hat{l}} \phi^{b} \phi^{a} \\ &+ \frac{1}{2} \left(- \left(T_{a\hat{r}} f^{\hat{r}}_{\hat{l}\hat{k}} + T_{p\hat{l}} f^{p}_{\hat{k}a} \right) T_{i\hat{\jmath}} + \left(T_{i\hat{r}} f^{\hat{r}}_{\hat{\jmath}\hat{k}} + T_{p(\hat{\jmath}} f^{p}_{\hat{k})i} \right) T_{a\hat{l}} \right) \phi^{\hat{\jmath}} \phi^{a} \,, \end{aligned} \tag{4.48}$$

where

$$\phi_i = A_i - i\, T_{i\hat{\jmath}} y^{\hat{\jmath}} \equiv T_{i\hat{\jmath}} \phi^{\hat{\jmath}} \,, \qquad \phi_{\hat{b}} = A_{\hat{b}} - i\, T_{\hat{b}a} x^a \equiv T_{\hat{b}a} \phi^a \,, \tag{4.49}$$

$a = \{k, l\}, \hat{b} = \{\hat{k}, \hat{l}\}$ and antisymmetrization in $\hat{k}, \hat{l}$ is understood.

Similarly to the type IIB case, (4.46) contains also terms that are not holomorphic in the brane moduli. These are proportional to combinations of structure constants that break supersymmetry. The $\mathcal{N} = 1$ Minkowski vacuum condition, $dJ_c = 0$, requires

$$T_{[k|\hat{r}} f^{\hat{r}}_{l]\hat{\jmath}} - T_{i\hat{\jmath}} f^{i}_{kl} = 0 \,, \quad T_{i[\hat{\jmath}|} f^{i}_{\hat{l}]k} - T_{k\hat{r}} f^{\hat{r}}_{\hat{\jmath}\hat{l}} = 0 \,, \quad T_{i\hat{\jmath}} f^{i}_{\hat{k}\hat{l}} \epsilon^{\hat{\jmath}\hat{k}\hat{l}} = 0 \,, \quad T_{i\hat{\jmath}} f^{\hat{\jmath}}_{kl} \epsilon^{ikl} = 0 \ . \tag{4.50}$$

Terms that are holomorphic in the brane moduli appear for example with the combination $T_{[k|\hat{r}} f^{\hat{r}}_{l]\hat{\jmath}} + T_{i\hat{\jmath}} f^{i}_{kl}$, while the combination with a minus sign gives rise to non holomorphic terms and is therefore discarded in (4.47).

We summarize in table 2 the torsion induced μ-terms for D6-branes on a factorizable torus, where the structure constants have one leg on each 2-torus. We have set the normalization of the matter fields to match the result for the D7-brane effective μ-term [35, 38] in vacua where a T-dual description is available.

5. Soft-terms on twisted tori

A background where supersymmetry is broken spontaneously by torsion or fluxes, induces soft-supersymmetry breaking terms on a D-brane living in it. In this section, we compute the pattern of soft-terms for factorizable twisted tori in the no-scale supersymmetry breaking vacua of sections 3.1 and 4.1.

	$D6_0$	$D6_1$
$t_1\mu_{11}$	$\frac{1}{2u_1}(T_2 f^{\hat{2}}_{\hat{1}3} - T_3 f^{\hat{3}}_{\hat{1}2} - 2T_1 f^1_{23})$	$\frac{1}{2s}(T_2 f^2_{\hat{3}\hat{1}} + T_3 f^3_{\hat{1}\hat{2}} - T_1 f^1_{\hat{2}\hat{3}})$
$t_2\mu_{22}$	$\frac{1}{2u_2}(-T_1 f^{\hat{1}}_{\hat{2}3} + T_3 f^{\hat{3}}_{\hat{2}1} + 2T_2 f^2_{31})$	$\frac{-1}{2u_3}(2T_3 f^3_{12} + T_1 f^{\hat{1}}_{2\hat{3}} + T_2 f^{\hat{2}}_{1\hat{3}})$
$t_3\mu_{33}$	$\frac{1}{2u_3}(T_1 f^{\hat{1}}_{\hat{3}2} - T_2 f^{\hat{2}}_{\hat{3}1} + 2T_3 f^3_{12})$	$\frac{1}{2u_2}(2T_2 f^2_{13} + T_1 f^{\hat{1}}_{3\hat{2}} + T_3 f^{\hat{3}}_{1\hat{2}})$

	$D6_2$	$D6_3$
$t_1\mu_{11}$	$\frac{1}{2u_3}(2T_3 f^3_{21} + T_1 f^{\hat{1}}_{2\hat{3}} + T_2 f^{\hat{2}}_{1\hat{3}})$	$\frac{-1}{2u_2}(2T_2 f^2_{31} + T_3 f^{\hat{3}}_{1\hat{2}} + T_1 f^{\hat{1}}_{3\hat{2}})$
$t_2\mu_{22}$	$\frac{1}{2s}(T_1 f^1_{\hat{2}\hat{3}} + T_2 f^2_{\hat{1}\hat{3}} + T_3 f^3_{\hat{1}\hat{2}})$	$\frac{1}{2u_1}(2T_1 f^1_{32} + T_3 f^{\hat{3}}_{2\hat{1}} + T_2 f^{\hat{2}}_{3\hat{1}})$
$t_3\mu_{33}$	$\frac{-1}{2u_1}(2T_1 f^1_{23} + T_2 f^{\hat{2}}_{3\hat{1}} + T_3 f^{\hat{3}}_{2\hat{1}})$	$\frac{1}{2s}(T_1 f^1_{\hat{2}\hat{3}} - T_2 f^2_{\hat{1}\hat{3}} - T_3 f^3_{\hat{1}\hat{2}})$

Table 2: Supersymmetric torsion induced μ-terms for $D6_0$ and $D6_i$ branes wrapping the cycles dual to $X^1X^2X^3$ and $\epsilon_{ijk}X^iY^{\hat{j}}Y^{\hat{k}}$ in a factorizable twisted torus.

Bulk and brane sectors combine in an $\mathcal{N} = 1$ supergravity. Brane moduli ϕ^i form $\mathcal{N} = 1$ chiral superfields charged under a non-Abelian gauge group (or just a U(1), for a single brane, which will be mostly the case for us). These couple to the neutral bulk moduli, such that brane fluctuations enter in the definition of the moduli descending from $\mathcal{N} = 2$ hypermultiplets. The Kähler potential is still given by (2.20), but ϕ^i enter in Π (2.19), and therefore modify the definition of the moduli (3.5), (4.8). These can be found in [33] and [75] respectively for D3-branes and D7-branes in SU(3) structure manifolds, while for the simplest case of factorizable toroidal models they are given in [76, 77]. If the gauge symmetry on the branes is unbroken, the vacuum expectation value of the fields ϕ^i vanishes and it is convenient to expand the Kähler potential in power series of ϕ^i

$$\begin{aligned} K(M,\bar{M},\phi,\bar{\phi}) &= \hat{K}(M,\bar{M}) + Z_{i\bar{j}}(M,\bar{M})\,\phi^i\bar{\phi}^{\bar{j}} + \frac{1}{2}\big(H_{ij}(M,\bar{M})\,\phi^i\phi^j + \text{h.c.}\big) + \cdots\,, \\ &\equiv \hat{K}(M,\bar{M}) + K_{Dp}(M,\bar{M},\phi,\bar{\phi}) \end{aligned} \tag{5.1}$$

where M denotes collectively bulk moduli. The function H_{ij} has been found to be nonzero for D3-branes [33] and for D7-branes in compactifications with an uplift to F-theory [78]. In the latter case they turn out to be equal to the $Z_{i\bar{j}}$ terms, $H_{ij} = Z_{i\bar{j}}$. The reason is that the D-brane moduli enter the Kähler potential through terms of the form $(\phi^i + \bar{\phi}^i)(\phi^j + \bar{\phi}^j)$.

Bulk and brane superpotential are also combined in the expansion

$$W(M,\phi) = \hat{W}(M) + W_{Dp}(M,\phi) = \hat{W}(M) + \frac{1}{2}\,\tilde{\mu}_{ij}(M)\,\phi^i\phi^j + \frac{1}{6}\,\tilde{Y}_{ijk}(M)\,\phi^i\phi^j\phi^k + \cdots\,. \tag{5.2}$$

The gauge couplings obey

$$g_{Dp}^{-2} = 2\,\mathrm{Re}\,f_{Dp}(M) \tag{5.3}$$

where f_{Dp} is the holomorphic gauge kinetic function. Inserting these in (2.29) and keeping terms up to cubic order in ϕ, we get an effective potential for the brane fields in the flat limit $M_{Pl} \to \infty$, $m_{3/2}$ fixed, of the form

$$V^{(\text{eff})} = Z^{i\bar{j}}\,(\partial_i W^{(\text{eff})})(\partial_{\bar{j}}\bar{W}^{(\text{eff})}) + \ m^2_{i\bar{j},\text{soft}}\,\phi^i\bar{\phi}^{\bar{j}} + \frac{1}{6}\,A_{ijk}\phi^i\phi^j\phi^k + \frac{1}{2}\,B_{ij}\phi^i\phi^j + \text{h.c.}\,, \tag{5.4}$$

where [27, 28, 79]

$$
\begin{aligned}
W^{(\mathrm{eff})} &= \frac{1}{2}\,\mu_{ij}\,\phi^i\phi^j + \frac{1}{3}\,Y_{ijk}\,\phi^i\phi^j\phi^k\,, \\
\mu_{ij} &= e^{\hat{K}/(2M_{Pl}^2)}\tilde{\mu}_{ij} + m_{3/2}H_{ij} - F^{\bar{I}}\bar{\partial}_{\bar{I}}H_{ij}\,, \\
Y_{ijk} &= e^{\hat{K}/(2M_{Pl}^2)}\tilde{Y}_{ijk}\;.
\end{aligned}
\tag{5.5}
$$

and the soft supersymmetry breaking terms read

$$
\begin{aligned}
m^2_{i\bar{\jmath},\mathrm{soft}} &= |m_{3/2}|^2\,Z_{i\bar{\jmath}} - F^I F^{\bar{J}} R_{I\bar{J}i\bar{\jmath}}, \\
A_{ijk} &= F^I D_I Y_{ijk}\,, \\
B_{ij} &= 2|m_{3/2}|^2\,H_{ij} - \bar{m}_{3/2}\bar{F}^{\bar{J}}\bar{\partial}_{\bar{J}}H_{ij} + m_{3/2}F^I D_I H_{ij} \\
&\quad - F^I\bar{F}^{\bar{J}}D_I\bar{\partial}_{\bar{J}}H_{ij} - e^{K/(2M_{Pl}^2)}\tilde{\mu}_{ij}\bar{m}_{3/2} + e^{K/(2M_{Pl}^2)}F^I D_I\tilde{\mu}_{ij}\,,
\end{aligned}
\tag{5.6}
$$

where

$$
\begin{aligned}
m_{3/2} &= e^{\hat{K}/(2M_{Pl}^2)}\frac{\hat{W}}{M_{Pl}^2}\,, & F^{\bar{I}} &= e^{\hat{K}/(2M_{Pl}^2)}\hat{K}^{\bar{I}J}D_J\hat{W}\,, \\
R_{I\bar{J}i\bar{\jmath}} &= \partial_I\bar{\partial}_{\bar{J}}Z_{i\bar{\jmath}} - \Gamma^k_{Ii}Z_{k\bar{l}}\Gamma^{\bar{l}}_{\bar{J}\bar{\jmath}}\,, & \Gamma^l_{Ii} &= Z^{l\bar{\jmath}}\partial_I Z_{\bar{\jmath}i}\,, \\
D_I Y_{ijk} &= \partial_I Y_{ijk} + \frac{1}{2M_{Pl}^2}\hat{K}_I Y_{ijk} - 3\Gamma^l_{I(i}Y_{jk)l}\,, \\
D_I\tilde{\mu}_{ij} &= \partial_I\tilde{\mu}_{ij} + \frac{1}{2M_{Pl}^2}\hat{K}_I\tilde{\mu}_{ij} - 2\Gamma^l_{I(i}\tilde{\mu}_{j)l}\;.
\end{aligned}
\tag{5.7}
$$

with M_{Pl} the 4d Planck mass. In these expressions we have taken the bulk moduli to be dimensionless (i.e. the quantum modes). This amounts to factor out the volume dependence of the 4d dilaton and the Kähler moduli into powers of M_{Pl}. The RR and NSNS forms have units of $(\text{length})^{-1}$, and $\hat{W} = M_{Pl}^3\tilde{W}$, $\hat{K} = M_{Pl}^2\tilde{K}$, with $\tilde{W}$ and $\tilde{K}$ respectively the dimensionless bulk superpotential and Kähler potential. In these units $\tilde{W}$ is a polynomial in the bulk moduli with integer coefficients.

Notice that for a D-brane superpotential of the form (5.2), the first term in (5.4) gives a "supersymmetric" mass term, as well as a trilinear C-coupling between two holomorphic and one antiholomorphic brane fields. These are given by

$$
m^2_{i\bar{\jmath},\mathrm{susy}} = e^{\hat{K}/M_{Pl}^2}\tilde{\mu}_{ik}\bar{\tilde{\mu}}_{\bar{l}\bar{\jmath}}Z^{k\bar{l}}\,, \quad C_{ij\bar{k},\mathrm{susy}} = e^{\hat{K}/M_{Pl}^2}\tilde{Y}_{ijl}\bar{\tilde{\mu}}_{\bar{m}\bar{k}}Z^{l\bar{m}}\;. \tag{5.8}
$$

Apart from these, additional mass terms and trilinear couplings are generated from H_{ij} through the Giudice-Masiero mechanism [65]

$$
\begin{aligned}
\mu_{ik,\mathrm{GM}} &= m_{3/2}H_{ik} - F^{\bar{I}}\bar{\partial}_{\bar{I}}H_{ik}\,, \\
m^2_{i\bar{\jmath},\mathrm{GM}} &= (\mu_{ik,\mathrm{GM}}\bar{\mu}_{\bar{l}\bar{\jmath},\mathrm{GM}} + \tilde{\mu}_{ik}\bar{\mu}_{\bar{l}\bar{\jmath},\mathrm{GM}} + \mu_{ik,\mathrm{GM}}\bar{\tilde{\mu}}_{\bar{l}\bar{\jmath}})Z^{k\bar{l}}\;.
\end{aligned}
\tag{5.9}
$$

The H_{ij} couplings can be thus absorbed into non-holomorphic contributions in the bulk moduli to the effective superpotential, $W^{(\mathrm{eff})}$, as already exposed in section 3.3.

In a generic compactification the total scalar masses for the brane fields therefore receive three tree-level contributions,

$$m^2_{i\bar{\jmath}} = m^2_{i\bar{\jmath},\text{susy}} + m^2_{i\bar{\jmath},\text{GM}} + m^2_{i\bar{\jmath},\text{soft}} \ . \tag{5.10}$$

However, the no-scale condition (2.30) often induces systematic cancellations which lead to vanishing $\mu_{ij,\text{GM}}$ and $m^2_{i\bar{\jmath},\text{soft}}$. More precisely, parameterizing $Z_{i\bar{\jmath}}$ and H_{ij} as

$$Z_{i\bar{\jmath}} = H_{ij} = \prod_I \frac{\text{const.}}{(M^I + \bar{M}^I)^{\alpha_I}} \ , \tag{5.11}$$

with M^I the collective bulk moduli, the condition to have $\mu_{ij,\text{GM}} = m^2_{i\bar{\jmath},\text{soft}} = 0$ is given by

$$\sum_{I,\ F^I \neq 0} \alpha_I = 1 \ . \tag{5.12}$$

In that case, it is easy to show additionally that B_{ij} gets no contribution from H_{ij}, i.e. the first four terms in the expression for B_{ij} in (5.6) also cancel.

The μ-terms computed in sections 3.3 and 4.3 are the total effective μ_{ij} of (5.5). In vacua where a T-dual description is available, they indeed match correctly the effective D7-brane μ-term computed in [35] by dimensional reduction of the DBI-CS action, and in [38] by analysis of the effective supergravity in compactifications of F-theory. Thus, in what follows we will not consider explicitly the H_{ij} couplings, but instead we will work in terms of $W^{(\text{eff})}$.

Before we move on to the specific supersymmetry breaking vacua, let us remark that all these are contributions coming from pure moduli mediation. As noticed in [80], non-perturbative or loop contributions such as anomaly mediation may be generically as important as moduli mediation contributions, and therefore in a concrete phenomenological model they should be taken into account.

5.1 Quaternionic breaking

We will compute soft-terms in the case of quaternionic breaking for D9 and D5-branes. The case of D6-branes in this type of supersymmetry breaking vacua can be easily obtained by T-duality. The gauge kinetic couplings and Kähler potential K_{D5}, K_{D9} for factorizable toroidal compactifications has been computed in [76, 77], obtaining up to second order in ϕ

$$\begin{aligned} K_{D9} &= \sum_i^3 \frac{|\phi^i|^2}{(U^i+\bar{U}^i)(T^i+\bar{T}^i)} \ , \quad & K_{D5_k} &= \sum_{i,j=1}^3 d_{ijk} \frac{|\phi^j|^2}{(T^i+\bar{T}^i)(U^j+\bar{U}^j)} + \frac{|\phi^k|^2}{(S+\bar{S})(U^k+\bar{U}^k)}, \\ f_{D9} &= S \, , & f_{D5_k} &= T^k \, , \end{aligned} \tag{5.13}$$

where the $D5_k$-branes are wrapping the k-th 2-torus and $d_{ijk} = 1$ for $i \neq j \neq k$, and 0 otherwise.

We consider no-scale vacua with supersymmetry spontaneously broken by the quaternionic sector. For that purpose we take the internal twisted torus to be a torus fibration over another torus, e.g. of the third torus over the first and second tori. The fibration is fully parameterized by the structure constants $f^{\bar{3}}_{\bar{1}\bar{2}}$, $f^{3}_{\bar{1}2}$, f^{3}_{12} and $f^{3}_{\bar{1}\bar{2}}$, with all the other structure constants zero. Assuming $G^+_{(1)}$ is independent of the Kähler modulus of the fiber, T^3 (i.e., $D_{T_3}W = 0$ and W is independent of S, T_1 and T_2), we obtain the gravitino mass,

$$m_{3/2} = M_{Pl} e^{\hat{K}/(2M_{Pl}^2)} (U^1 + \overline{U}^1)(U^2 + \overline{U}^2)(T^3 + \overline{T}^3) f^{3}_{\bar{1}\bar{2}}. \tag{5.14}$$

and the scalar potentials for the light scalar modes of the D-branes in the no-scale vacuum are,[13]

$$\begin{aligned} V_{D9} &= \frac{e^{\hat{K}/M_{Pl}^2}}{M_{Pl}^4} \left| M_{Pl}^2 \partial_{\phi^3} W_{D9} + (\phi^3)^* \hat{W} \right|^2 , \\ V_{D5_1} &= \frac{e^{\hat{K}/M_{Pl}^2}}{M_{Pl}^4} |M_{Pl}^2 \partial_{\phi^3} W_{D5_1} + (\phi^2)^* \hat{W}|^2 , \\ V_{D5_2} &= \frac{e^{\hat{K}/M_{Pl}^2}}{M_{Pl}^4} |M_{Pl}^2 \partial_{\phi^1} W_{D5_2} + (\phi^1)^* \hat{W}|^2 , \\ V_{D5_3} &= 0 , \end{aligned} \tag{5.15}$$

for pure moduli mediation.

We have summarized in table 3 the pattern of soft supersymmetry-breaking terms which results of plugging (3.4) and (3.47) into the above scalar potentials. For that we have assumed the usual superpotential trilinear couplings, $\tilde{Y}_{ijk} = \epsilon_{ijk}$. The rescaled structure constants, $\tilde{f}^I_{JK}$, are defined in (3.46). The resulting pattern is clearly related to the one arising in the worldvolume of D3 and D7-branes in the dual compactification [32–35]. Indeed, T-dualizing along the third torus, $D5_3$-branes are mapped to D3-branes, whereas $D5_1$, $D5_2$ and $D9$-branes are mapped respectively to $D7_2$, $D7_1$ and $D7_3$-branes. As expected, the light modes of $D5_3$-branes remain massless, whereas only one complex geometric moduli of the $D5_1$, $D5_2$ and $D9$-branes becomes massive, corresponding to the geometric moduli of the dual D7-brane. This structure of zero modes can be understood in terms of the condition (5.12). Indeed, making use of (5.13), we get that $\mu_{ij,\mathrm{GM}} = m^2_{i\bar{j},\mathrm{soft}} = 0$ for all the scalars in the $D5_3$-branes, and all the scalars but ϕ^1, ϕ^2 and ϕ^3 in the $D5_2$, $D5_1$ and $D9$-branes respectively, in agreement with the results of table 3.

Both in the supersymmetric and in the no-scale cases, the couplings induced by $\mathcal{W}_3$ (i.e., by the structure constants $f^{\bar{3}}_{\bar{1}\bar{2}}$, $f^{3}_{\bar{1}2}$) give rise to masses and C-terms satisfying (5.8) and therefore are compatible with $\mathcal{N}=1$ supersymmetry with some massive chiral supermultiplets. On the other hand, $\mathcal{W}_1$, proportional to $f^{3}_{\bar{1}\bar{2}}$, gives rise to couplings satisfying

$$\mathrm{Tr}(m^2_{i,\mathrm{soft}}) = m^2_{3/2} \quad , \quad A_{ijk} = h_{ijk} \mathrm{Tr}\ (m^2_{i,\mathrm{soft}}) , \tag{5.16}$$

with, $h_{ijk} = e^{\hat{K}/2M_{Pl}} \epsilon_{ijk} (Z_{i\bar{i}} Z_{j\bar{j}} Z_{k\bar{k}})^{-1/2}$ the physical Yukawa. This behavior was already observed in [32–35] for D3 and D7-branes in the presence of 3-form fluxes. D6-branes in vacua where supersymmetry is broken by the quaternionic sector follow the same pattern of soft supersymmetry breaking terms.

[13] We take the usual rescaling of the matter fields, $\phi^i \to (Z_{i\bar{i}})^{-1/2} \phi^i$, in order to have canonically normalized kinetic terms.

	$D9$	$D5_1$	$D5_2$	$D5_3$
μ_{11}	0	0	$4e^{\hat{K}/2}\tilde{f}^{3}_{1\bar{2}}t_3$	0
μ_{22}	0	$4e^{\hat{K}/2}\tilde{f}^{3}_{\bar{1}2}t_3$	0	0
μ_{33}	$4e^{\hat{K}/2}\tilde{f}^{\bar{3}}_{\bar{1}\bar{2}}t_3$	0	0	0
$m^2_{1\bar{1}}$	0	0	$\lvert\mu_{33}\rvert^2+\lvert m_{3/2}\rvert^2$	0
$m^2_{2\bar{2}}$	0	$\lvert\mu_{22}\rvert^2+\lvert m_{3/2}\rvert^2$	0	0
$m^2_{3\bar{3}}$	$\lvert\mu_{11}\rvert^2+\lvert m_{3/2}\rvert^2$	0	0	0
B_{11}	0	0	$2\mu_{33}\bar{m}_{3/2}$	0
B_{22}	0	$2\mu_{22}\bar{m}_{3/2}$	0	0
B_{33}	$2\mu_{11}\bar{m}_{3/2}$	0	0	0
A_{123}	$g_{D9}m_{3/2}$	$g_{D5_1}m_{3/2}$	$g_{D5_2}m_{3/2}$	0
$C_{1\bar{2}\bar{3}}$	0	0	$\mu_{33}\,g_{D5_2}$	0
$C_{\bar{1}2\bar{3}}$	0	$\mu_{22}\,g_{D5_1}$	0	0
$C_{\bar{1}\bar{2}3}$	$\mu_{11}\,g_{D9}$	0	0	0

Table 3: Torsion induced soft parameters for $D9$, $D5_1$, $D5_2$ and $D5_3$-branes, in a no-scale vacuum of a factorizable twisted torus with W independent of S, T_1, T_2, and $D_M W = 0$ for the remaining moduli. The gauge coupling constants are $g_{D9} = (S+\bar{S})^{-1/2}$ and $g_{D5_k} = (T^k + \bar{T}^k)^{-1/2}$, and we have set $M_{Pl} = 1$.

The pattern of moduli mediated soft supersymmetry-breaking terms therefore can be recast in terms of a small set of parameters: the gravitino mass plus some topological μ-terms for each stack of branes. Hence, consider for example the no-scale $K3 \times T^2$ fibration of section 3.2. In a complex basis the structure constants (3.25) read

$$f^{3}_{\bar{1}2} = f^{3}_{1\bar{2}} = -f^{\bar{3}}_{\bar{1}\bar{2}} = \frac{1+iu}{4u^2}\ . \tag{5.17}$$

From (5.15) we then obtain the tree-level scalar potentials for the D-brane fields

$$\begin{aligned}
V_{D9} &= \left|\frac{\phi^1\phi^2}{\sqrt{2s}} - \frac{(1+iu)\phi^3 + t(3iu+1)(\phi^3)^*}{(32\tilde{t}_1\tilde{t}_2 t u^3)^{1/2}}\right|^2 \\
V_{D5_1} &= \left|\frac{\phi^1\phi^3}{\sqrt{2\tilde{t}_1}} - \frac{(1+iu)\phi^2 - t(3iu+1)(\phi^2)^*}{(32\tilde{t}_1\tilde{t}_2 t u^3)^{1/2}}\right|^2 \\
V_{D5_2} &= \left|\frac{\phi^2\phi^3}{\sqrt{2\tilde{t}_1}} + \frac{(1+iu)\phi^1 - t(3iu+1)(\phi^1)^*}{(32\tilde{t}_1\tilde{t}_2 t u^3)^{1/2}}\right|^2 \\
V_{D5_3} &= 0
\end{aligned} \tag{5.18}$$

More detailed phenomenological analysis for this class of vacua in the D3/D7 setup, taking into account other effects such as non-perturbative effects or α' corrections to the Kähler potential, can be found in [80 − 83].

5.2 Mixed breaking

For type IIA we have seen in section 4.1.2 that there is another class of no-scale vacua where supersymmetry is broken by moduli belonging in part to descendants of $N = 2$

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hypermultiplets, and in part to descendants of scalars in vector multiplets. In this section we compute the pattern of soft-terms for D6-branes placed on this type of vacua in factorizable (twisted) T^6 models, with 3 complex structure and 3 Kähler moduli, and structure constants with one leg on each torus. We consider a no-scale vacua where supersymmetry is broken by T_1, U_2, U_3, i.e. $\partial_{T_1} W = \partial_{U_2} W = \partial_{U_3} W = 0$ in the vacuum, while $D_{T_2} W = D_{T_3} W = D_{U_1} W = D_S W = 0$. Supersymmetry breaking is due solely to the torsion in this class of vacua, i.e. H and all RR fluxes are zero. The bulk superpotential is

$$\hat{W} = M_{Pl}^2 e^{-\hat{K}/(2M_{Pl}^2)} m_{3/2} = M_{Pl}^3 \left[S T_2 f^2_{\hat{1}\hat{3}} - S T_3 f^3_{\hat{1}\hat{2}} + U_1 T_2 f^{\hat{2}}_{\hat{1}3} - U_1 T_3 f^{\hat{3}}_{\hat{1}2} \right] \tag{5.19}$$

The other structure constants allowed in a factorizable torus vanish in this type of vacua.

As explicitly shown in the particular example of section 4.2.2, the imaginary parts of the Kähler moduli appearing in the superpotential, Im T_2 and Im T_3, are related to the background $\overline{H}$. Indeed, from $H = 0$ we get

$$\overline{H} = -dB = -(f^2_{\hat{1}\hat{3}} \mathrm{Im}\ T_2 - f^3_{\hat{1}\hat{2}} \mathrm{Im}\ T_3) e^4 \wedge e^5 \wedge e^6 + (f^{\hat{2}}_{\hat{1}3} \mathrm{Im}\ T_2 - f^{\hat{3}}_{\hat{1}2} \mathrm{Im}\ T_3) e^4 \wedge e^2 \wedge e^3\ . \tag{5.20}$$

Since $\overline{H} \neq 0$ may induce additional μ-terms in the D6-branes which we are not computing here, in what follows we set $\overline{H} = \mathrm{Im}\ T_2 = \mathrm{Im}\ T_3 = 0$. The final result however should not depend on this choice, as the VEV for physical field, H, is fixed.

The Kähler potential for D6-branes is the T-dual version of (5.13), where we should exchange Kähler and complex structure moduli, $D9$ by $D6_0$ and $D5_k$ by $D6_k$. We get

$$\begin{aligned} K_{D6_0} &= \sum_i^3 \frac{|\phi^i|^2}{(U^i+\bar{U}^i)(T^i+\bar{T}^i)}\,, & K_{D6_k} &= \sum_{i,j=1}^3 d_{ijk} \frac{|\phi^j|^2}{(U^i+\bar{U}^i)(T^j+\bar{T}^j)} + \frac{|\phi^k|^2}{(S+\bar{S})(T^k+\bar{T}^k)}\,, \\ f_{D6_0} &= S\,, & f_{D6_k} &= U^k\ . \end{aligned} \tag{5.21}$$

Rescaling the matter fields as in the quaternionic breaking (see footnote 13), we get the following potential for D6-branes in these vacua up to cubic order in ϕ^i,

$$\begin{aligned} V_{D6_0} &= \frac{e^{\hat{K}/M_{Pl}^2}}{M_{Pl}^4} \left| M_{Pl}^2 \partial_{\phi^1} W_{D6_0} \right|^2\,, \\ V_{D6_1} &= \frac{e^{\hat{K}/M_{Pl}^2}}{M_{Pl}^4} \left| M_{Pl}^2 \partial_{\phi^1} W_{D6_1} \right|^2\,, \\ V_{D6_2} &= \frac{e^{\hat{K}/M_{Pl}^2}}{M_{Pl}^4} \left[\left| M_{Pl}^2 \partial_{\phi^2} W_{D6_2} + (\phi^2)^* \hat{W} \right|^2 + \left| M_{Pl}^2 \partial_{\phi^3} W_{D6_2} + (\phi^3)^* \hat{W} \right|^2 \right. \\ &\qquad \left. + \left| M_{Pl}^2 \partial_{\phi^1} W_{D6_2} - (\phi^1)^* \hat{W} \right|^2 - 2 |\phi^1 \hat{W}|^2 \right]\,, \\ V_{D6_3} &= \frac{e^{\hat{K}/M_{Pl}^2}}{M_{Pl}^4} \left[\left| M_{Pl}^2 \partial_{\phi^2} W_{D6_3} + (\phi^2)^* \hat{W} \right|^2 + \left| M_{Pl}^2 \partial_{\phi^3} W_{D6_3} + (\phi^3)^* \hat{W} \right|^2 \right. \\ &\qquad \left. + \left| M_{Pl}^2 \partial_{\phi^1} W_{D6_3} - (\phi^1)^* \hat{W} \right|^2 - 2 |\phi^1 \hat{W}|^2 \right]\ . \end{aligned} \tag{5.22}$$

We have summarized in table 4 the pattern of soft supersymmetry-breaking terms which results of plugging (4.6), (4.47) and (4.48) into the above scalar potentials, for mixed

	$D6_0$	$D6_1$	$D6_2$	$D6_3$
μ_{11}	$2e^{\frac{\tilde{K}}{2}}(t_2 f^{\hat{2}}_{\hat{1}3} - t_3 f^{\hat{3}}_{\hat{1}2})$	$2e^{\frac{\tilde{K}}{2}}(t_2 f^{2}_{\hat{3}\hat{1}} + t_3 f^{3}_{\hat{1}\hat{2}})$	0	0
μ_{22}	0	0	$2e^{\frac{\tilde{K}}{2}}(t_2 f^{2}_{\hat{1}\hat{3}} + t_3 f^{3}_{\hat{1}\hat{2}})$	$2e^{\frac{\tilde{K}}{2}}(t_2 f^{\hat{2}}_{3\hat{1}} + t_3 f^{\hat{3}}_{2\hat{1}})$
μ_{33}	0	0	$2e^{\frac{\tilde{K}}{2}}(t_2 f^{2}_{\hat{1}3} + t_3 f^{\hat{3}}_{\hat{1}2})$	$2e^{\frac{\tilde{K}}{2}}(t_2 f^{2}_{\hat{3}\hat{1}} - t_3 f^{3}_{\hat{1}\hat{2}})$
$m^2_{1\bar{1}}$	$\lvert\mu_{11}\rvert^2$	$\lvert\mu_{11}\rvert^2$	$-\lvert m_{3/2}\rvert^2$	$-\lvert m_{3/2}\rvert^2$
$m^2_{2\bar{2}}$	0	0	$\lvert\mu_{22}\rvert^2 + \lvert m_{3/2}\rvert^2$	$\lvert\mu_{22}\rvert^2 + \lvert m_{3/2}\rvert^2$
$m^2_{3\bar{3}}$	0	0	$\lvert\mu_{33}\rvert^2 + \lvert m_{3/2}\rvert^2$	$\lvert\mu_{33}\rvert^2 + \lvert m_{3/2}\rvert^2$
B_{11}	0	0	0	0
B_{22}	0	0	$2\mu_{22}\,\bar{m}_{3/2}$	$2\mu_{22}\,\bar{m}_{3/2}$
B_{33}	0	0	$2\mu_{33}\,\bar{m}_{3/2}$	$2\mu_{33}\,\bar{m}_{3/2}$
A_{123}	0	0	$g_{D6_2} m_{3/2}$	$g_{D6_3} m_{3/2}$
$C_{1\bar{2}\bar{3}}$	$\mu_{11}\, g_{D6_0}$	$\mu_{11}\, g_{D6_1}$	0	0
$C_{\bar{1}2\bar{3}}$	0	0	$\mu_{22}\, g_{D6_2}$	$\mu_{22}\, g_{D6_3}$
$C_{\bar{1}\bar{2}3}$	0	0	$\mu_{33}\, g_{D6_2}$	$\mu_{33}\, g_{D6_3}$

Table 4: Torsion induced soft parameters for $D6_M$-branes, in a no-scale vacuum of a factorizable twisted torus with W independent of T_1, U_2, U_3. The gauge coupling constants are $g_{D6_0} = (S + \bar{S})^{-1/2}$ and $g_{D6_k} = (U^k + \bar{U}^k)^{-1/2}$, and we have set $M_{Pl} = 1$.

supersymmetry breaking vacua. Note that, unlike the case for quaternionic breaking, there is at least one modulus that becomes massive for each type of brane. This confirms the fact that this class of vacua is not related by T-duality to the quaternionic one.

Assuming (5.11) for H_i, and making use of (5.12) and (5.21), it is possible to check that $\mu_{ij,\mathrm{GM}} = m^2_{i\bar{j},\mathrm{soft}} = 0$ for all the scalars in the worldvolume of the $D6_0$ and $D6_1$-branes. Hence, the μ-terms for these branes, shown in table 4, correspond to purely supersymmetric (holomorphic in bulk moduli) $\tilde{\mu}$-terms. Moreover, $\mathrm{Re}\,\mathcal{W}_1 \sim m_{3/2}$ gives rise to soft couplings in the worldvolume of the $D6_2$ and $D6_3$-branes which satisfy the relations (5.16). To this regard, the induced soft masses for ϕ^1 are always tachyonic, signaling an instability of the $D6_2$ and $D6_3$-branes at the origin, within this type of vacua. It is tempting to identify this instability with a Higgs mechanism. The final state, however, is not captured by the potentials (5.22), as they were derived under the assumption $\langle \phi^i \rangle = 0$. Analogous tachyonic masses were obtained in heterotic compactifications with asymmetric Kähler domination [85].[14] It would be desirable to obtain a better understanding of the nature of these tachyonic modes within this context.

Finally, as an illustration of how the above equations apply in a concrete model, consider the example of section (4.2.2). The non trivial structure constants can be read from (4.34),

$$f^{2}_{\hat{3}\hat{1}} = f^{3}_{\hat{1}\hat{2}} = f^{\hat{2}}_{3\hat{1}} = f^{\hat{3}}_{\hat{1}2} = 1 \; . \tag{5.23}$$

[14] We are grateful to Luis Ibáñez for this observation.

From (5.22) then we obtain the following tree-level scalar potentials for the D-brane moduli,

$$
\begin{aligned}
V_{D6_0} &= \left| \frac{\phi^2\phi^3}{\sqrt{2s}} - \frac{\phi^1}{s(8t_1u_2u_3)^{\frac{1}{2}}} \right|^2 , \\
V_{D6_1} &= \left| \frac{\phi^2\phi^3}{\sqrt{2s}} + \frac{\phi^1}{s(8t_1u_2u_3)^{\frac{1}{2}}} \right|^2 , \\
V_{D6_2} &= \left| \frac{\phi^1\phi^2}{\sqrt{2u_2}} - \frac{(\phi^3)^*}{(8t_1u_2u_3)^{\frac{1}{2}}} \right|^2 + \left| \frac{\phi^1\phi^3}{\sqrt{2u_2}} + \frac{(\phi^2)^*}{(8t_1u_2u_3)^{\frac{1}{2}}} \right|^2 + \left| \frac{\phi^2\phi^3}{\sqrt{2u_2}} + \frac{(\phi^1)^*}{(8t_1u_2u_3)^{\frac{1}{2}}} \right|^2 - \frac{|\phi^1|^2}{4t_1u_2u_3}, \\
V_{D6_3} &= \left| \frac{\phi^1\phi^2}{\sqrt{2u_3}} - \frac{(\phi^3)^*}{(8t_1u_2u_3)^{\frac{1}{2}}} \right|^2 + \left| \frac{\phi^1\phi^3}{\sqrt{2u_3}} + \frac{(\phi^2)^*}{(8t_1u_2u_3)^{\frac{1}{2}}} \right|^2 + \left| \frac{\phi^2\phi^3}{\sqrt{2u_3}} + \frac{(\phi^1)^*}{(8t_1u_2u_3)^{\frac{1}{2}}} \right|^2 - \frac{|\phi^1|^2}{4t_1u_2u_3} .
\end{aligned}
\tag{5.24}
$$

6. Conclusions

We have explored the conditions to have no-scale supergravity vacua on orientifolds of SU(3) structure manifolds, with supersymmetry spontaneously broken at tree-level. Although we have covered a broad set of supergravity backgrounds, we have found only two classes of solutions, depending on whether the supersymmetry breaking is mediated by neutral matter descending from $\mathcal{N}=2$ hypermultiplets or from a mixture of vector and hypermultiplets. The first case, which we have denoted "quaternionic breaking", corresponds to T-duals of the known type IIB no-scale vacua with 3-form fluxes, and is fully characterized by a single ISD poly-form mixing fluxes and torsion. The second, labelled as "mixed breaking", is instead related to fluxless Scherk-Schwarz compactifications and can be characterized by a purely imaginary poly-form.

We have also computed the effective μ-terms induced by the torsion of the SU(3) structure manifold in the gauge theory of D5, D6 and D9-branes, for vacua based on twisted tori. These encode the tree-level dynamics of the branes in the supergravity vacuum. The resulting patterns for type IIB (IIA) vacua, summarized in tables 1 and 2, can be nicely arranged in terms of the holomorphic (symplectic) properties of the structure constants. A similar fact was already observed in [35, 38] for the D7-brane flux induced μ-term. The present patterns, however, contain a much richer structure, allowing for mass terms for mostly all the brane moduli. The potential applications for model building are therefore promising.

Notice that, due to the presence of flat directions, every attempt of extracting phenomenological information from these vacua should also take the quantum dynamics into account. In this sense, the patterns of soft terms for pure moduli mediation presented in section 5 are partial and, in a concrete phenomenological model, should be completed with non-perturbative and loop contributions.

Since a full string theory treatment of non-perturbative effects is missing, one is usually advocated to implement those at the level of the effective field theory. From this perspective, the structure of μ-terms turns out to be also determinant, as the non-perturbative dynamics is constrained by the number of fermionic zero modes.

No-scale solutions of ten dimensional supergravity have been considered very frequently in the framework of type IIB Calabi-Yau orientifolds with O3/O7-planes and 3-form fluxes.

In this case, the supersymmetry is often restored when non-perturbative effects are present. To this regard, we expect a similar behavior for the full no-scale quaternionic breaking family of vacua. It would be very interesting however to extend this analysis to the case of mixed breaking studied here, and to check in particular if the breaking of supersymmetry is actually propagated to the complete solution. It would also be nice to understand the tachyonic instability observed for one of the brane moduli in this family of no-scale vacua.

Finally, there are also other directions which we believe may deserve further research. The conditions for supersymmetric vacua allow for more general structures, such as SU(3)× SU(3). It is natural to expect that these solutions also admit non-supersymmetric marginal deformations analogous to the ones discussed here. It may be interesting to look for new families of no-scale vacua within this context. Understanding the structure of the effective supergravity is a major task for phenomenological applications of string theory. We hope to come back soon to these issues.

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A. Conventions and spinors

We take orientation conventions for which

$$*J = \frac{1}{2} J \wedge J\,, \qquad \int J \wedge J \wedge J > 0\,. \tag{A.1}$$

J and Ω can be obtained from the SU(3) invariant spinor η and the metric by

$$\begin{aligned} \eta_+^\dagger \gamma^m \eta_\pm &= 0\,, & \eta_-^\dagger \gamma^{mnp} \eta_+ &= \tfrac{1}{2} i \left(\frac{\mathcal{N}_J}{\mathcal{N}_\Omega}\right)^{\frac{1}{2}} \Omega^{mnp}\,, \\ \eta_\pm^\dagger \gamma^{mn} \eta_\pm &= \pm\tfrac{1}{2} i\, J^{mn}\,, & \eta_+^\dagger \gamma^{mnp} \eta_- &= \tfrac{1}{2} i \left(\frac{\mathcal{N}_J}{\mathcal{N}_\Omega}\right)^{\frac{1}{2}} \bar{\Omega}^{mnp}\,, \end{aligned} \tag{A.2}$$

where $\mathcal{N}_J$ and $\mathcal{N}_\Omega$ are given in (2.3) and $\eta_\pm^\dagger \eta_\pm = \frac{1}{2}$, $\eta_+^* = \eta_-$ (i.e. we are using the intertwiner between γ_m and $-\gamma_m^*$ to be 1).

The Mukai pairing between forms is related to the norm of bispinors by [10],

$$\int \langle \Phi, \chi \rangle = \frac{1}{2} \mathrm{tr} \left(i \gamma_7 \Phi_\epsilon^T \chi_\epsilon \right) \mathcal{N}_J\,, \tag{A.3}$$

where $\Phi_\epsilon, \chi_\epsilon$ are the bispinors corresponding to the forms Φ, χ. We find also convenient to use the Fierz identity

$$\eta_+ \tilde{\eta}_\pm^\dagger = \frac{1}{4} \sum_{k=0}^{6} \frac{1}{k!} \left(\tilde{\eta}_\pm^\dagger \gamma_{m_1 \ldots m_k} \eta_+ \right) \gamma^{m_k \ldots m_1}\,, \tag{A.4}$$

to write the forms in (2.8).

B. Decomposition in SU(3) representations

Part of the underlying approach that we use in the paper relies on the decomposition of forms in SU(3) representations. For poly-forms, it is more convenient to use the generalized Hodge diamond [43, 84, 45], whose elements are given by the different poly-forms in (4.3). Each component is then computed by an appropriate integral. Concretely, the different components of the 3-form decomposition (3.3) are obtained from,

$$G^{+}_{(1)} = -\frac{i}{12\mathcal{N}_J}\int \Omega\wedge G\,, \quad G^{-}_{(1)} = \frac{i}{12\mathcal{N}_J}\int \bar{\Omega}\wedge G\,, \quad G^{\pm}_{(3)} = \frac{1}{2}J\llcorner G^{\pm}\,. \tag{B.1}$$

Analogously, the components of the even form G in the SU(3) decomposition (4.3) are expressed in terms of the following integrals,

$$\begin{aligned}
G^{\pm}_{(1)} &= \pm\frac{i}{8\mathcal{N}_\Omega}\int\langle e^{\mp iJ},G\rangle\,, & G^{\pm}_{mn} &= \pm\frac{i}{32\mathcal{N}_J}J_{mp}J_{nq}\int\langle\gamma^p e^{\pm iJ}\gamma^q,G\rangle\,,\\
G^{+}_{m} &= -\frac{1}{16\mathcal{N}_\Omega}J_{mn}\int\langle\gamma^n\Omega,G\rangle\,, & \tilde{G}^{+}_{m} &= \frac{1}{16\mathcal{N}_\Omega}J_{mn}\int\langle\overline{\Omega}\gamma^n,G\rangle\,,\\
G^{-}_{m} &= -\frac{1}{16\mathcal{N}_\Omega}J_{mn}\int\langle\gamma^n\overline{\Omega},G\rangle\,, & \tilde{G}^{-}_{m} &= \frac{1}{16\mathcal{N}_\Omega}J_{mn}\int\langle\Omega\gamma^n,G\rangle\,,
\end{aligned} \tag{B.2}$$

with γ^m given in (4.4).

For real single-degree even forms, we use also the following SU(3) decomposition,

$$\begin{aligned}
F_2 &= \frac{1}{3}F_2^{(1)}\,J + \mathrm{Re}\,(F_2^{(3)}\llcorner\overline{\Omega}) + F_2^{(8)}\,,\\
F_4 &= \frac{1}{6}F_4^{(1)}J\wedge J + \mathrm{Re}\,(F_4^{(3)}\wedge\overline{\Omega}) + F_4^{(8)}\,,\\
F_6 &= \frac{1}{6}F_6^{(1)}\,J\wedge J\wedge J\,.
\end{aligned} \tag{B.3}$$

These singlets are a combination of the four singlets $G^{\pm}_{(1)}$, $G^{\pm}_{mn}J^{mn}$ defined in (4.3).

Finally, for F_3 and H, we use

$$F_3 = \frac{3\mathcal{N}_J}{\mathcal{N}_\Omega}\mathrm{Re}(F_{(1)}\bar{\Omega}) + F_{(3)}\wedge J + F_{(6)}\,, \tag{B.4}$$

$$H = \frac{3\mathcal{N}_J}{\mathcal{N}_\Omega}\mathrm{Re}(H_{(1)}\bar{\Omega}) + H_{(3)}\wedge J + H_{(6)}\,. \tag{B.5}$$

where comparing to (3.3), $F_{(1)} = F^{+}_{(1)} = (F^{-}_{(1)})^*$. In O6 compactifications H is odd under the orientifold action, same as $\mathrm{Im}\,\Omega$. This implies that $H^{(1)}$ is real.

C. Torsion classes on twisted tori

For completeness in this appendix we present the torsion classes for a twisted torus in terms of the structure constants f^a_{bc} defined in (3.41). For alternative expressions, the reader may also consult [16].

Defining the spin connection 1-form with holomorphic indices, ω^{mn}, through,

$$dz^m + \omega^{mn} \wedge z_n + \omega^{m\bar{n}} \wedge \bar{z}_n = 0\,, \tag{C.1}$$

with holomorphic vectors $z^m = e^m + iU^m{}_n e^n$, for $m = 1, 2, 3$, and acting with the exterior derivative on Ω and J given in (3.43), we extract the torsion classes,

$$\mathcal{W}_1 = \frac{2i}{3}\epsilon_{mno}\imath_{z^m}\omega^{no}\,, \tag{C.2}$$

$$\mathcal{W}_2 = -\epsilon_{mno}\omega^{mn} \wedge z^o - \mathcal{W}_1 J\,, \tag{C.3}$$

$$\mathcal{W}_3 = \frac{i}{2}\omega_{mn} \wedge z^m \wedge z^n + \frac{3}{4i}\frac{\mathcal{N}_J}{\mathcal{N}_\Omega}\overline{\mathcal{W}}_1\Omega \;+\; c.c.\,, \tag{C.4}$$

$$\mathcal{W}_4 = \mathcal{W}_5 = 0\,, \tag{C.5}$$

with $\epsilon_{123} = -i$. In terms of (3.41) the spin connection reads,

$$\omega^{ab} \equiv -\frac{1}{2}\left(\imath_{e^a}de^b - \imath_{e^b}de^a - e_c(\imath_{e^a}\imath_{e^b}de^c)\right) = \frac{1}{2}\left(f^b_{cd}e^c g^{ad} - f^a_{cd}e^c g^{bd} - f^c_{de}g^{ad}g^{be}e_c\right)\,, \tag{C.6}$$

with $e_c \equiv g_{bc}e^b$. Hence, in terms of structure constants with holomorphic/antiholomorphic indices,

$$\mathcal{W}_1 = \frac{i}{3}g^{m\bar{r}}g^{n\bar{s}}\epsilon_{mno}f^o_{\bar{r}\bar{s}}\,, \tag{C.7}$$

$$\mathcal{W}_2 = -\mathcal{W}_1 J + \epsilon_{mno}g^{n\bar{s}}\left(f^o_{\bar{p}\bar{s}} + \frac{g_{q\bar{p}}}{2}f^q_{\bar{o}\bar{s}}\right) z^m \wedge \bar{z}^p\,, \tag{C.8}$$

$$\mathcal{W}_3 = \frac{i}{2}\left(g_{m\bar{s}}f^{\bar{s}}_{n\bar{o}} - \frac{g_{r\bar{o}}}{2}f^r_{mn}\right) z^m \wedge z^n \wedge \bar{z}^o + c.c.\,, \tag{C.9}$$

$$\mathcal{W}_4 = \mathcal{W}_5 = 0\ . \tag{C.10}$$

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